Taxation of Unknown Beneficial Owners: Treaty Shopping with Incomplete Information

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Abstract

This paper examines treaty shopping in a game-theoretic model with incomplete information. An investor has private information about her country of residence and income at source and chooses a direct or indirect investment route across national borders to minimize tax. A tax agency has private information about its audit cost and decides whether to audit the investor. An audit is costly but it can generate additional revenue for the tax agency. If the audit reveals that the investor chose an indirect route, the tax agency imposes a penalty tax on the investor. Under reasonable assumptions, I show that no pure-strategy Bayesian Nash equilibrium exists and then construct an equilibrium where the tax agency of a low cost type audits the investor with positive probability and the investor of a country with tax-minimizing indirect routes chooses such indirect routes with positive probability. I also compute the decrease in the tax agency’s expected payoff due to indirect routing, i.e., treaty shopping, and show the payoff dominance of the equilibrium random audit strategy to the pure always-audit strategy. Under foreign tax credit systems in residence countries with relatively high tax rates, there is an equilibrium where the investor chooses the direct route regardless of her type, and thus, treaty shopping can be prevented in equilibrium.

JEL classification: H250, H870, K340

Keywords: treaty shopping, incomplete information, tax-minimizing route, Bayesian Nash equilibrium, random audit, foreign tax credit

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1 Introduction

A beneficial owner generally refers to a person entitled to the benefits of a tax treaty, such as the application of reduced withholding tax rates on dividends, interest, and royalties. This person must be a resident of a contracting state of the tax treaty. However, in cases of tax treaty shopping, it turns out that a beneficial owner is not a true resident of the contracting state. In such cases, an investor operates an intermediary (conduit) entity in a state with a favorable tax treaty to obtain those tax treaty benefits, even if she is not a resident of that state.\(^1\) When dealing with such cases, a tax agency may not know the true residency of the investor. In other words, the investor has private information about her country of residence.

Tax treaty shopping is considered to be an improper use of tax treaties.\(^2\) The purpose of this paper is to examine tax treaty shopping in a game-theoretic model with incomplete information. I would like to address the following questions: How frequently do investors use tax-minimizing indirect routes for tax treaty shopping? How should tax authorities counteract? How much does tax treaty shopping affect tax revenue in a source country? Can tax treaty shopping be prevented by tax relief rules, such as foreign tax credit systems?

Let us imagine the following situation. An investor, individual or corporate, lives in country \(h\) and plans to invest in country \(s\). From this invest-

\(^{1}\)For instance, a US multinational company, Corning, invested in a Korean manufacturer, Samsung Corning, through a Hungarian subsidiary, Corning Hungary Data Services Kft., where “Kft.” is a form of a Limited Liability Company (LLC) in Hungary. According to the Korea-US tax treaty, withholding tax rates on dividends are set at 10 or 15 percent depending on percentages of shares. However, the Korea-Hungary tax treaty sets the minimum withholding tax rate at 5 percent, and in Hungary, there is no withholding tax on dividends paid to non-residents.

\(^{2}\)OECD (2013, 2014) highlights the issues on treaty abuse in the Base Erosion and Profit Shifting (BEPS) project as follows: “Treaty abuse is one of the most important sources of BEPS concerns. The Commentary on Article 1 of the OECD Model Tax Convention already includes a number of examples of provisions that could be used to address treaty-shopping situations as well as other cases of treaty abuse, which may give rise to double non-taxation. Tight treaty anti-abuse clauses coupled with the exercise of taxing rights under domestic laws will contribute to restore source taxation in a number of cases.”
ment, the investor expects to earn income \( m \) as dividends in country \( s \) and will repatriate her income to country \( h \). The investor’s type is determined by her country of residence and income at source, i.e., by a pair of \((h, m)\). The investor of type \((h, m)\) intends to minimize tax when she remits income \( m \) to country \( h \). The investor of type \((h, m)\) can choose a direct route or an indirect route, which incorporates intermediary entities established in other countries.

Tax treaties, as well as national tax laws, determine tax rates on cross-border payments. These tax rates determine taxes paid along an investment route. The investor’s net-of-tax income also depends on tax relief rules in her residence country. First it is assumed that residence countries have deduction or exemption systems. Foreign tax credit systems will be considered later.

In country \( s \), a tax agency is informed of the inbound investment. The tax agency can audit the investor by incurring the audit cost \( c \), which determines the tax agency’s type. In other words, the tax agency of type \( c \) can choose whether to audit the investor. While an audit is costly, it can generate additional revenue for the tax agency. If the audit reveals that the investor chose an indirect route, the tax agency imposes a penalty tax on the investor. However, if the investor chose the direct route, the tax agency does not gain additional revenue from the audit but incurs the audit cost.

Both the tax agency and the investor have private information about their own types. The tax agency knows its own type \( c \) but the investor may not know the tax agency’s type. Meanwhile, the investor knows her own type \((h, m)\) but the tax agency may not know the investor’s type. The tax agency and the investor have subjective beliefs about each other’s type. These beliefs are described by probability distributions \( \pi \) and \( \phi \).

In a model with two possible residence countries and two possible audit cost types, I show that no pure-strategy Bayesian Nash equilibrium exists. Thus, in equilibrium, it is necessary for the tax agency and the investor to choose mixed strategies. I construct a Bayesian Nash equilibrium where the tax agency of the low audit cost type audits the investor with positive prob-
ability and the investor of the country with tax-minimizing indirect routes chooses such indirect routes with positive probability. In this equilibrium, the tax agency’s audit probability is calculated as the ratio of tax rate difference to expected penalty tax rate with respect to the investor’s beliefs $\phi$. The investor’s indirect-routing probability is calculated as the ratio of audit cost to expected penalty tax with respect to the tax agency’s beliefs $\pi$. However, in the equilibrium, the tax agency of the high audit cost type chooses not to audit at all, and the investor of the country with a tax-minimizing direct route chooses the direct route for sure.

In the equilibrium, the tax agency earns the same expected payoff regardless of its type, while the investor earns different payoffs depending on her type. The investor’s equilibrium payoffs are equal to the payoffs when the investor always chooses the direct route. I also calculate the decrease in the tax agency’s expected payoff due to indirect routing, i.e., treaty shopping. This payoff loss can be calculated with the average tax rate spread and the ratio of the audit cost to the penalty tax rate.

Interestingly, if the penalty tax rate is set sufficiently high, the tax agency earns a greater expected payoff by choosing the equilibrium random audit strategy than by always auditing the investor. However, in any case, the investor earns the same expected payoff.

Under foreign tax credit systems in the residence countries, if domestic tax rates are higher than or equal to tax treaty rates, there is an equilibrium where the investor chooses the direct route regardless of her type. Otherwise, there is an equilibrium where the investor chooses tax-minimizing indirect routes with positive probability. This equilibrium is similar to the equilibrium under deduction or exemption systems.

Treaty shopping can be prevented by foreign tax credit systems in residence countries with relatively high tax rates. The cost of this prevention is borne by residence countries, in terms of tax credits given to investors, whereas the benefit is enjoyed by source countries, in terms of increased tax revenue. Without overcoming such free-rider problems between countries, it
may be difficult to sustain foreign tax credit systems in residence countries. Recently, France, Germany, Japan, and the United Kingdom moved away from tax credit systems to introduce exemption for foreign-source dividends.

The contribution of this paper is threefold.

Broadly, this paper contributes to the economics of international taxation. In the literature, a central theme has been tax competition between countries, which can choose tax rates, as well as tax relief rules, such as foreign tax credit, deduction, and exemption systems. Another important issue is to examine theoretical and empirical relations between tax treaties and Foreign Direct Investment (FDI). Chisik and Davies (2004) study the effects of FDI on tax treaty bargaining. Recently, increasing attention has been paid to international tax policies to deal with issues of Base Erosion and Profit Shifting (BEPS). Dharmapala (2014) provides a survey of empirical studies to assess the magnitude of BEPS. In contrast, this paper analyzes the effects of tax treaty rates, tax relief systems, and private information on strategic interactions between investors and tax authorities.

More specifically, this paper contributes to the literature on tax treaty shopping and the ownership structures of multinational firms. This literature uses firm-level data to examine factors, such as withholding taxes, affecting ownership structures. Mintz and Weichenrieder (2010) show that there is a growing number of German firms organizing indirect ownership structures for foreign subsidiaries and suggest that treaty shopping is a main reason for indirect structures. Lewellen and Robinson (2013) show a similar finding

\[3\text{For related studies, see Bond and Samuelson (1989), Janeba (1995), Konan (1997), and Davies (2003).}\]

\[4\text{Conversely, Blonigen and Davies (2004) examine the effects of tax treaties on FDI stocks in the United States, and discover substantial heterogeneity in treaty effects across countries. Blonigen et al. (2014) find that US multinational firms using differentiated inputs show increased sales of foreign affiliates in countries with new tax treaties.}\]


\[6\text{Weyzig (2013) also confirms that reductions in withholding tax rates on dividends are significant determinants of FDI indirectly routed through the Netherlands. Lejour (2014) provides similar findings based on data of OECD countries.}\]

5
for US firms. However, these studies are based on empirical approaches. This paper develops a game-theoretic model to better understand strategic decision-making in the context of treaty shopping.

This paper also contributes to the theory of auditing. This literature has developed principal-agent models to examine optimal audit design. For instance, Mookherjee and Png (1989) analyze a model where a risk-averse agent has private information about her income and reports it to a risk-neutral principal who then chooses to audit the agent at a cost. In their model, random audits turn out to be optimal because random audits can reduce expected costs without significantly distorting the agent’s incentive to report truthfully. Border and Sobel (1987) also show the efficiency of random auditing in a model with a risk-neutral agent. These studies only deal with auditing against underreported income in a single jurisdiction. In contrast, this paper examines random audits against tax planning techniques in a multi-jurisdictional setting.

The rest of this paper is organized as follows. Section 2 develops a game-theoretic model with incomplete information. Section 3 analyzes Bayesian Nash equilibria in this model and examines the merit of the equilibrium random audit strategy. Section 4 discusses the effects of tax relief systems on treaty shopping. Section 5 concludes.

2 Model

Let $N = \{1, 2, 3, \ldots, n\}$ denote the set of all countries. An investor, individual or corporate, plans to invest in country $s \in N$ but lives in country $h \in H \subseteq N \setminus \{s\}$. From this investment, the investor will earn income $m \in M \subseteq \mathbb{R}_{++}$ as dividends in country $s$ and repatriate her income to country $h$. Country $s$ is called the source country while country $h$ is called the residence (or home) country. The investor’s type is denoted by a pair

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7Reinganum and Wilde (1988) show that a tax agency may gain in terms of revenue and compliance by increasing taxpayer uncertainty regarding its audit cost. In contrast, Reinganum and Wilde (1985) develop a model where random audit rules are weakly dominated by audit cutoff rules.
$(h, m) \in H \times M$. The investor of type $(h, m)$ intends to minimize tax when she remits income $m$ to country $h$, i.e., the investor intends to maximize her net-of-tax income in the residence country.

An investment route, or simply a route, is defined as a series of countries, $h, i, \ldots, j, s$, from country $h$ to country $s$. Given a route $h, i, \ldots, j, s$, countries $i$ through $j$ are called pass-through countries. A strategy of the investor, denoted by $b(\cdot)$, is a function mapping each type $(h, m)$ to a route $b(h, m)$. If $b(h, m) = h, i, \ldots, j, s$, the investor of type $(h, m)$ chooses an indirect route to invest in country $s$ by establishing entities in countries $i$ through $j$ and making her investment indirectly through these entities. If $b(h, m) = h, s$, she chooses the direct route to invest in country $s$. When the investor remits her income from country $s$ to country $h$, the remittance route follows the reverse order of the countries in the investment route.

In the source country a tax agency is informed of the inbound investment. By incurring the audit cost $c \in C \subseteq \mathbb{R}^+$, the tax agency can audit the investor. The tax agency’s type is determined by the audit cost $c \in C$. A strategy of the tax agency, denoted by $a(\cdot)$, is a function mapping each type $c$ to an action $a(c) \in \{0, 1\}$. If $a(c) = 1$, the tax agency of type $c$ audits the investor. Otherwise, the tax agency does not.

The investor has private information about her country of residence and income at source, i.e., the investor knows her own type $(h, m) \in H \times M$ but the tax agency may not know the investor’s type. The tax agency has subjective beliefs about the investor’s type, described by a probability distribution $\pi$ over $H \times M$.

The tax agency has private information about its audit cost, i.e., the tax agency knows its own type $c \in C$ but the investor may not know the tax agency’s type. The investor has subjective beliefs about the tax agency’s type, described by a probability distribution $\phi$ over $C$.

Both the tax agency and the investor know a tax rate $t_i$ in country $i$ and a tax rate $t_{ij}$ on dividends under the tax treaty between countries $i$ and $j$. A withholding tax is imposed by country $i$ at the tax treaty rate $t_{ij}$ when an
entity in country \( i \) remits dividends to another entity in country \( j \). A tax rate matrix is given as follows:

\[
T = \begin{bmatrix}
t_{11} & t_{12} & \cdots & t_{1n} \\
t_{21} & t_{22} & \cdots & t_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
t_{n1} & t_{n2} & \cdots & t_{nn}
\end{bmatrix}
\]

Let \( w(b(h, m)) \) denote the withholding tax rate of a route \( b(h, m) \) at source such that \( w(b(h, m)) = t_{sh} \) if \( b(h, m) = h, s \) and \( w(b(h, m)) = t_{sj} \) if \( b(h, m) = h, i, \ldots, j, s \). Let \( f(b(h, m)) \) denote the foreign tax rate of a route \( b(h, m) \) such that \( f(b(h, m)) = t_{sh} \) if \( b(h, m) = h, s \) and \( f(b(h, m)) = 1 - (1 - t_{sj})(1 - t_{j})(1 - t_{ih})(1 - t_{ih})(1 - t_{ih}) \) if \( b(h, m) = h, i, \ldots, j, s \).

When the investor of type \( (h, m) \) chooses a route \( b(h, m) \), the net-of-tax income is \( m(1 - f(b(h, m)))(1 - t_h) \) in country \( h \), and the tax revenue is \( mw(b(h, m)) \) in country \( s \). Here it is assumed that residence countries have either deduction or exemption systems. The tax rate in country \( h \) is \( t_h > 0 \) under deduction systems and \( t_h = 0 \) under exemption systems. Foreign tax credit systems will be considered later.

Let \( p^* \in \mathbb{R}_{++} \) and let \( p(a(c), b(h, m)) \) denote the penalty tax rate of an action \( a(c) \) and a route \( b(h, m) \) such that \( p(a(c), b(h, m)) = p^* \) if \( a(c) = 1 \) and \( b(h, m) = h, i, \ldots, j, s \) and \( p(a(c), b(h, m)) = 0 \) otherwise. If the investor of type \( (h, m) \) chooses an indirect route \( b(h, m) = h, i, \ldots, j, s \) and the tax agency of type \( c \) audits the investor, i.e., \( a(c) = 1 \), the tax agency imposes the penalty tax \( mp^* \), which the investor must pay to the tax agency. Otherwise, there is no penalty tax. The tax agency of type \( c \) incurs the audit cost \( ca(c) \).

To summarize, given a probability distribution \( \pi \) and a type \( c \), the payoff function of the tax agency (or player A) can be written as follows:

\[
u_A(a(\cdot), b(\cdot); c) = E_\pi [mw(b(\cdot)) + mp(a(\cdot), b(\cdot)) - ca(c)]
\]

Given a probability distribution \( \phi \) and a type \( (h, m) \), the payoff function of the investor (or player B) can be written as follows:

\[
u_B(a(\cdot), b(\cdot); h, m) = E_\phi [m(1 - f(b(h, m)))(1 - t_h) - mp(a(\cdot), b(h, m))]
\]
When choosing whether to audit the investor, the tax agency does not know the investor’s type and action. The tax agency only knows its own type, i.e., the audit cost. When choosing an investment route, the investor does not know the tax agency’s type and action. The investor only knows her own type \((h,m)\), i.e., her country of residence and income at source. The tax agency and the investor play a game with incomplete information.

3 Analysis

In this section I examine (Bayesian Nash) equilibria. For each \(h \in H\), let \(R(h)\) denote the set of all routes from country \(h\) to country \(s\). Let \(R(H) = \bigcup_{h \in H} R(h)\). For each \(h \in H\), a route \(b^* \in R(h)\) is tax-minimizing if for each \(b \in R(h)\), \(f(b^*) \leq f(b)\). Let \(H^D\) denote the set of residence countries from which direct routes are tax-minimizing. Let \(H^I = H \setminus H^D\) denote the set of residence countries with no tax-minimizing direct routes. For each \(h \in H^I\), there is a tax-minimizing indirect route \(b^* = h,i,\ldots,j,s\) with \(f(b^*) < t_{sh}\).

Let us consider cases with two possible home countries. Let \(H = \{1,2\}\) with \(H^I = \{1\}\) and \(H^D = \{2\}\). Let \(M = \{m_1, m_2\}\). Let \(C = \{c_L, c_H\}\) with \(c_L < c_H\). The tax agency’s beliefs are described by a probability distribution \(\pi\) with \(\pi(1,m_1) = \pi_{11}\), \(\pi(1,m_2) = \pi_{12}\), \(\pi(2,m_1) = \pi_{21}\), and \(\pi(2,m_2) = \pi_{22}\). Let \(\bar{m} = (\pi_{11} + \pi_{12})E_\pi[m|h = 1] = \pi_{11}m_1 + \pi_{12}m_2\), where \(E_\pi[m|h = 1]\) denotes the conditional expectation of income \(m\) given \(h = 1\), i.e., earned by the investor of type \((1,m)\). The investor’s beliefs are described by a probability distribution \(\phi\) with \(\phi(c_L) = \phi_L\) and \(\phi(c_H) = \phi_H\).

It is assumed that the high audit cost is greater than the maximum expected penalty tax, i.e., \(p^*E_\pi[m] < c_H\), and that the low audit cost is smaller than the expected penalty tax when indirect routes are chosen only by the investor of type \((1,m)\), i.e., \(c_L < mp^*\). Also, the penalty tax rate discounted by the probability of the low audit cost is greater than the difference in rates of net-of-tax income, i.e., \((t_{s1} - f(b^*)) (1 - t_1) < \phi_L p^*\). Later I will discuss the cases when each of these assumptions is not satisfied.

First I show that no pure-strategy equilibrium exists.
Theorem 1. There is no pure-strategy equilibrium.

The proof of Theorem 1 proceeds in four steps.

Step 1. For the tax agency of type \( c_H \), it is dominated to audit the investor, i.e., to choose \( a(c_H) = 1 \). Let \( a^*(c_H) = 0 \) and \( a(c_H) = 1 \). For each strategy \( b(\cdot) \) of the investor, \( u_A(a^*(c_H), b(\cdot); c_H) = E_\pi [mw(b(\cdot))] \). Also, \( u_A(a(c_H), b(\cdot); c_H) = E_\pi [mw(b(\cdot))] + E_\pi [mp(a(c_H), b(\cdot))] - c_H \). Because \( E_\pi [mp(a(c_H), b(\cdot))] \leq p^* E_\pi [m] < c_H \), it holds that \( u_A(a(c_H), b(\cdot); c_H) < u_A(a^*(c_H), b(\cdot); c_H) \).

Step 2. Because \( a(c) \in \{0, 1\} \) for \( c \in \{c_L, c_H\} \), there are four pure strategies for the tax agency. However, from Step 1, pure strategies with \( a(c_H) = 1 \) are dominated, and thus, not played in equilibrium. There remain two pure strategies that may constitute an equilibrium. Let \( a^0(\cdot) \) denote the strategy such that \( a^0(c_L) = 0 \) and \( a^0(c_H) = 0 \). Let \( a^1(\cdot) \) denote the strategy such that \( a^1(c_L) = 1 \) and \( a^1(c_H) = 0 \).

Step 3. Suppose that the tax agency chooses \( a^0(\cdot) \). Given \( a^0(\cdot) \), to maximize her expected payoff, the investor chooses a tax-minimizing indirect route when she is of type \((1, m)\) and chooses the direct route when she is of type \((2, m)\). Let \( b^0(\cdot) \) denote such a strategy. Because \( b^0(\cdot) \) is the best response to \( a^0(\cdot) \), \((a^0(\cdot), b^0(\cdot))\) is the only strategy profile that can be an equilibrium. Given \( b^0(\cdot) \), if the tax agency chooses \( a^0(\cdot) \), for type \( c_L \), \( u_A(a^0(\cdot), b^0(\cdot); c_L) = E_\pi [mw(b^0(\cdot))] \). If the tax agency chooses \( a^1(\cdot) \), for type \( c_L \), \( u_A(a^1(\cdot), b^0(\cdot); c_L) = E_\pi [mw(b^0(\cdot))] + E_\pi [mp(1, b^0(\cdot))] - c_L \). Because \( E_\pi [mp(1, b^0(\cdot))] = \bar{m} p^* \) and \( c_L < \bar{m} p^* \), it holds that \( u_A(a^0(\cdot), b^0(\cdot); c_L) < u_A(a^1(\cdot), b^0(\cdot); c_L) \). Thus, \( a^0(\cdot) \) is not a best response to \( b^0(\cdot) \), and \((a^0(\cdot), b^0(\cdot))\) is not an equilibrium.

Step 4. Suppose that the tax agency chooses \( a^1(\cdot) \). Given \( a^1(\cdot) \), to maximize her expected payoff, the investor always chooses the direct route regardless of her type. This is because \((t_{s_1} - f(b^*))(1 - t_1) < \phi_L p^* \) implies that, for the investor of type \((1, m)\), \( u_B(a^1(\cdot), b^*; 1, m) = m(1 - f(b^*))(1 - t_1) - \phi_L m p^* < m(1 - t_{s_1})(1 - t_1) = u_B(a^1(\cdot), 1, s; 1, m) \). For the investor of type \((2, m)\), the...
direct route is tax-minimizing with no risk of the penalty tax. Let \( b_1(\cdot) \) denote the strategy that the investor always chooses the direct route. Because \( b_1(\cdot) \) is the best response to \( a_1(\cdot) \), \( (a_1(\cdot), b_1(\cdot)) \) is the only strategy profile that can be an equilibrium. Given \( b_1(\cdot) \), if the tax agency chooses \( a_1(\cdot) \), for type \( c_L \), the expected payoff is \( u_A(a_1(\cdot), b_1(\cdot); c_L) = E_\pi[mt_{sh}] - c_L \). If the tax agency chooses \( a_0(\cdot) \), for type \( c_L \), the expected payoff is \( u_A(a_0(\cdot), b_1(\cdot); c_L) = E_\pi[mt_{sh}] \). Thus, \( a_1(\cdot) \) is not a best response to \( b_1(\cdot) \), and \( (a_1(\cdot), b_1(\cdot)) \) is not an equilibrium. □

Theorem 1 implies that in equilibrium it is necessary for the players to choose mixed strategies. Thus, I construct a mixed-strategy equilibrium. In this equilibrium, the tax agency of type \( c_L \) audits the investor with positive probability, calculated as the ratio of tax rate difference to expected penalty tax rate with respect to the investor’s beliefs \( \phi \). The investor of type \((1, m)\) chooses tax-minimizing indirect routes with positive probability, calculated as the ratio of audit cost to expected penalty tax with respect to the tax agency’s beliefs \( \pi \).

**Theorem 2.** There is an equilibrium where (i) the tax agency of type \( c_L \) audits the investor with probability \( (t_{s1} - f(b^*)) \frac{(1 - t_1)}{\phi_L p^*} \), (ii) the tax agency of type \( c_H \) chooses no audit for sure, (iii) the investor of type \((1, m)\) chooses tax-minimizing indirect routes with probability \( c_L / \bar{m} p^* \), and (iv) the investor of type \((2, m)\) chooses the direct route for sure.

The proof of Theorem 2 proceeds in five steps.

**Step 1.** For the tax agency of type \( c_H \), it is dominant to choose not to audit. Let \( a^*(c_H) = 0 \) and \( a(c_H) = 1 \). For each strategy \( b(\cdot) \) of the investor, \( u_A(a^*(c_H), b(\cdot); c_H) = E_\pi[\bar{m} w(b(\cdot))] \). Also, \( u_A(a(c_H), b(\cdot); c_H) = E_\pi[\bar{m} w(b(\cdot))] + E_\pi[mp(a(c_H), b(\cdot))] - c_H \). Because \( E_\pi[mp(a(c_H), b(\cdot))] \leq p^* E_\pi[\bar{m}] < c_H \), it holds that \( u_A(a(c_H), b(\cdot); c_H) < u_A(a^*(c_H), b(\cdot); c_H) \).

**Step 2.** For the investor of type \((2, m)\), it is weakly dominant to choose the direct route. For each \( m \in M \), let \( b^*(2, m) = 2, s \) be the direct route.
Because country 2 belongs to $H^D$, the direct route is tax-minimizing, i.e., for each $b \in R(2)$, $f(b^*(2,m)) \leq f(b)$. For each $b \in R(2)$ with $b \neq b^*(2,m)$, if $a(c) = 0$, then $p(a(c),b^*(2,m)) = p(a(c),b) = 0$ and $u_B(a(c),b;2,m) \leq u_B(a(c),b^*(2,m);2,m)$. If $a(c) = 1$, then $p(a(c),b^*(2,m)) = 0 < p^* = p(a(c),b)$ and $u_B(a(c),b;2,m) < u_B(a(c),b^*(2,m);2,m)$.

**Step 3.** For the investor of type $(1, m)$, it is dominated to choose an indirect route that is not tax-minimizing. For $m \in M$, let $b(1,m) = b$ denote such a route. Because country 1 belongs to $H^I$, there is a tax-minimizing indirect route $b^* = 1, i, \ldots, j, s$ with $f(b^*) < t_{s1}$. Let $b^*(1,m) = b^*$. Because $b$ and $b^*$ are indirect routes, for each $a(c) \in \{0, 1\}$, $p(a(c),b) = p(a(c),b^*)$. Because $b^*$ is tax-minimizing but $b$ is not, $f(b^*) < f(b)$. Thus, for each $a(c) \in \{0, 1\}$ and each $m \in M$, it holds that $u_B(a(c),b;1,m) < u_B(a(c),b^*;1,m)$.

**Step 4.** Suppose that there are $\ell \geq 1$ tax-minimizing indirect routes from country 1 to country $s$. Each of such routes is denoted by $b^k = 1, i_k, \ldots, j_k, s$ with $k = 1, \ldots, \ell$, and is played with probability $q_k$, where $\sum_{k=1}^{\ell} q_k = c_L/\bar{m}p^*$. Let $(a^*(\cdot), b^*(\cdot))$ denote the strategy profile specified as follows:

$$
a^*(c) = \begin{cases} 1 & \text{with probability } (t_{s1} - f(b^*))(1-t_1)/\phi_Lp^* \text{ for } c = c_L \\ 0 & \text{with probability } 1 - (t_{s1} - f(b^*))(1-t_1)/\phi_Lp^* \text{ for } c = c_H \\ 0 & \text{with probability 1} \text{ for } c = c_H
\end{cases}
$$

$$
b^*(h, m) = \begin{cases} b^k & \text{with probability } q_k \text{ for } h = 1 \\ 1, s & \text{with probability } 1 - \sum_{k=1}^{\ell} q_k \text{ for } h = 1 \\ 2, s & \text{with probability 1} \text{ for } h = 2
\end{cases}
$$

Given the investor’s strategy $b^*(\cdot)$, it holds that $E_{\pi}[mp(1,b^*(\cdot))] = c_L$ and $u_A(1,b^*(\cdot);c_L) = E_{\pi}[mw(b^*(\cdot))]$. Also, $u_A(0,b^*(\cdot);c_L) = E_{\pi}[mw(b^*(\cdot))]$.

Thus, the tax agency of type $c_L$ is indifferent between $a^*(c_L) = 1$ and $a^*(c_L) = 0$. Given the tax agency’s strategy $a^*(\cdot)$, for $m \in M$, it holds that $u_B(a^*(\cdot),b^k;1,m) = m(1-t_{s1})(1-t_1) = u_B(a^*(\cdot),1,s;1,m)$. Thus, the investor of type $(1, m)$ is indifferent between $b^*(1,m) = b^k$ and $b^*(1,m) = 1, s$.

**Step 5.** From Step 1, it is dominant for the tax agency of type $c_H$ to choose no audit. Regardless of what the investor does, the tax agency of type $c_H$
will choose no audit, as specified in \(a^*(\cdot)\). From Step 2, it is weakly dominant for the investor of type \((2, m)\) to choose the direct route. Thus, regardless of what the tax agency does, the investor of type \((2, m)\) will choose the direct route, as specified in \(b^*(\cdot)\). Because it is dominated for the investor of type \((1, m)\) to choose an indirect route that is not tax-minimizing, from Step 3, the investor of type \((1, m)\) will only choose either the direct route or a tax-minimizing indirect route. By choosing the mixed strategy specified in Step 4, each player makes the other player indifferent between the actions played with positive probability. Therefore, \((a^*(\cdot), b^*(\cdot))\) is an equilibrium. □

Next I calculate equilibrium expected payoffs and the decrease in the tax agency’s expected payoff due to the use of indirect routes for treaty shopping.

**Remark 1.** In equilibrium \((a^*(\cdot), b^*(\cdot))\), the tax agency of type \(c\) earns an expected payoff of \(u_A(a^*(\cdot), b^*(\cdot); c) = E_\pi [mw(b^*(\cdot))]\). Thus, the tax agency earns the same expected payoff, regardless of its type. Meanwhile, the investor of type \((h, m)\) earns an expected payoff of \(u_B(a^*(\cdot), b^*(\cdot); h, m) = m(1 - t_{sh})(1 - t_h)\), which depends on the investor’s type.

In Remark 1, it is worthwhile to note that the investor’s equilibrium payoffs are equal to the payoffs when the investor always chooses the direct route. In contrast, the tax agency’s equilibrium payoff is smaller than the payoff when the investor chooses the direct route and the tax agency chooses no audit. By comparing these payoffs I calculate the payoff loss of the tax agency due to treaty shopping.

**Remark 2.** If the investor always chooses the direct route regardless of her type, the tax agency earns an expected payoff of \(E_\pi [mt_{sh}]\) by choosing no audit. However, in equilibrium \((a^*(\cdot), b^*(\cdot))\), the tax agency’s expected payoff is calculated as \(E_\pi [mw(b^*(\cdot))] = E_\pi [mt_{sh}] - \bar{m} \sum_{k=1}^\ell q_k(t_{s1} - t_{sj_k})\), where \(j_k\) is the last pass-through country in each tax-minimizing route \(b^k = 1, i_k, \ldots, j_k, s\) that is played with probability \(q_k\). Therefore, the tax agency’s expected payoff decreases by \(\bar{m} \sum_{k=1}^\ell q_k(t_{s1} - t_{sj_k})\), as the investor chooses tax-minimizing indirect routes for treaty shopping.
In Remark 2, the payoff loss of the tax agency is calculated with the weighted sum of tax rate spreads between the direct route and tax-minimizing indirect routes with weights given by the investor’s equilibrium probability. Given \( \ell \geq 1 \) tax-minimizing indirect routes, if each of such routes is played with equal probability, i.e., if \( q_k = c_L/\ell \bar{m} p^* \) for \( k = 1, \ldots, \ell \), the payoff loss can be rewritten as \( (c_L/p^*)(1/\ell) \sum_{k=1}^{\ell} (t_{s1} - t_{sjk}) \), which is calculated with the average tax rate spread and the ratio of the low audit cost to the penalty tax rate.

Now I examine whether the tax agency gains from the equilibrium random audit strategy.

**Theorem 3.** If the penalty tax rate is greater than the maximum tax rate spread, the tax agency earns a greater expected payoff by choosing the equilibrium random audit strategy than by always auditing the investor, while the investor earns the same expected payoff.

**Proof.** Suppose that the penalty tax rate is greater than the maximum tax rate spread, i.e., if \( p^* > \max_k (t_{s1} - t_{sjk}) \). If the tax agency must audit regardless of its type, the investor will always chooses the direct route to maximize her expected payoff. In this case, the tax agency of type \( c \) earns \( E_\pi [mt_{sh}] - c \), and the investor of type \( (h,m) \) earns \( m(1 - t_{sh})(1 - t_h) \). However, if the tax agency chooses the equilibrium random audit strategy, in the equilibrium, the tax agency of type \( c \) earns \( E_\pi [mt_{sh}] - \bar{m} \sum_{k=1}^{\ell} q_k (t_{s1} - t_{sjk}) \), and the investor of type \( (h,m) \) earns \( m(1 - t_{sh})(1 - t_h) \). Because \( p^* > \max_k (t_{s1} - t_{sjk}) \) and \( \sum_{k=1}^{\ell} q_k = c_L/\bar{m} p^* \), it holds that \( c_L = \bar{m}(c_L/\bar{m} p^*) \max_k (t_{s1} - t_{sjk}) = \bar{m} \sum_{k=1}^{\ell} q_k \max_k (t_{s1} - t_{sjk}) \geq \bar{m} \sum_{k=1}^{\ell} q_k (t_{s1} - t_{sjk}) \). Therefore, the tax agency earns a greater expected payoff by choosing the equilibrium random audit strategy than by always auditing the investor. However, in both cases, the investor earns the same expected payoff. □

Contrary to classical studies on random audits, e.g., Border and Sobel (1987) and Mookherjee and Png (1989), in this model, the investor’s tax avoidance technique is not based on underreporting but on indirect routing.
Even if the investor reports her income truthfully, if she uses tax planning techniques, such as indirect routing, the tax agency can choose random audit strategies with sufficiently high penalty tax rates to earn greater revenue while maintaining the same expected payoff for the investor.

**Remark 3.** If the tax agency of type $c_L$ audits the investor and the tax agency of type $c_H$ does not, the investor will always choose the direct route to maximize her expected payoff. In this case, the tax agency of type $c_L$ earns $E_\pi[mt_{sh}] - c_L$, the tax agency of type $c_H$ earns $E_\pi[mt_{sh}]$, and the investor of type $(h,m)$ earns $m(1 - t_{sh})(1 - t_h)$. Thus, the tax agency of type $c_H$ earns less by choosing the equilibrium random audit strategy. As assumed in Theorem 3, if the penalty tax rate is greater than the maximum tax rate spread, the tax agency of type $c_L$ earns more by choosing the equilibrium random audit strategy. Otherwise, the tax agency of type $c_L$ may also earn less. In any case, the investor earns the same expected payoff.

**Remark 4.** Consider the cases when the aforementioned assumptions are not satisfied. If $p^*E_\pi[m] \geq c_H$, the tax agency of type $c_H$ may have the incentive to audit the investor. In this case, if the investor always chooses indirect routes, auditing is profitable for the tax agency, regardless of its type. If $c_L \geq \bar{mp^*}$, the tax agency of type $c_L$ may have no incentive to audit the investor. In this case, if the investor of type $(1,m)$ chooses indirect routes and the investor of type $(2,m)$ chooses the direct route, auditing is not profitable for the tax agency. If $(t_{m1} - f(b^*)) (1 - t_1) \geq \phi_L p^*$, the investor of type $(1,m)$ may have the incentive to choose a tax-minimizing indirect route $b^*$ even when audited by the tax agency.

### 4 Discussion

In this section I consider foreign tax credit systems in residence countries and discuss the effects of tax relief systems on treaty shopping. There are three different tax relief systems that affect the net-of-tax income of investors in their home countries. Under deduction systems, foreign taxes are deducted as
costs from taxable income before taxes are imposed by home countries. Under exemption systems, certain items of foreign-source income, such as dividends, are exempt from home country taxation. Under tax credit systems, foreign taxes are credited against tax liabilities in home countries.

In the model it is assumed that home countries have either deduction or exemption systems. Under deduction systems in country $h$, $t_h > 0$. Under exemption systems in country $h$, $t_h = 0$.

Under tax credit systems in country $h$, the investor’s net-of-tax income is determined by the greater of the foreign tax rate $f(b(h, m))$ and the domestic tax rate $t_h$. The payoff function of the investor is given as follows:

$$u_B(a(\cdot), b(\cdot); h, m) = m(1 - \max\{f(b(h, m)), t_h\}) - E_\phi [mp(a(\cdot), b(h, m))]$$

If $f(b(h, m)) \leq t_h$, foreign taxes are no greater than domestic tax liabilities, and thus, the investor pays $m(t_h - f(b(h, m)))$ in country $h$ and $mf(b(h, m))$ in countries along the investment route $b(h, m)$. The investor’s net-of-tax income is $m(1 - t_h)$. However, if $t_h < f(b(h, m))$, foreign taxes are greater than domestic tax liabilities, and thus, the investor pays no tax in country $h$. The investor’s net-of-tax income is $m(1 - f(b(h, m)))$.

Let us consider cases with the two residence countries analyzed in the previous section. The following results show how domestic tax rates affect equilibrium behavior. The proofs are presented in the Appendix.

**Theorem 4.** Under tax credit systems in the residence countries, if $t_{s1} \leq t_1$, there is an equilibrium where the tax agency chooses no audit and the investor chooses the direct route regardless of their types.

**Theorem 5.** Under tax credit systems in the residence countries, if $t_1 < t_{s1}$, there is an equilibrium where (i) the tax agency of type $c_L$ audits the investor with probability $(t_{s1} - t_f)/\phi_Lp^*$ such that $t_f = \max\{f(b^*), t_1\}$, (ii) the tax agency of type $c_H$ chooses no audit for sure, (iii) the investor of type $(1, m)$ chooses tax-minimizing indirect routes with probability $c_L/\bar{m}p^*$, and (iv) the investor of type $(2, m)$ chooses the direct route for sure.
Remark 5. When $t_{s1} \leq t_1$, in the equilibrium of Theorem 4, the tax agency of type $c$ earns an expected payoff of $E_\pi [mt_{sk}]$, regardless of its type. The investor of type $(1, m)$ earns $m(1 - t_1)$ while the investor of type $(2, m)$ earns $m(1 - \max\{t_{s2}, t_2\})$. In this equilibrium, no treaty shopping occurs, and the tax agency of the source country earns the maximum possible tax revenue without incurring the audit cost. However, the residence countries earn tax revenues reduced by foreign tax credits given to the investor.

Remark 6. When $t_1 < t_{s1}$, in the equilibrium of Theorem 5, the tax agency of type $c$ earns an expected payoff of $E_\pi [mwb^*(\cdot)]$, regardless of its type. This is the same payoff as in the equilibrium of Theorem 2 and Remark 1. Also, the investor of type $(1, m)$ earns $m(1 - t_{s1})$ and the investor of type $(2, m)$ earns $m(1 - \max\{t_{s2}, t_2\})$. In the source country the tax agency earns less tax revenue due to treaty shopping. Moreover, the residence countries earn zero tax revenue when foreign tax credits exceed domestic tax liabilities.

Under tax credit systems with $t_{s1} \leq t_1$, it is a weakly dominant strategy for the investor to choose the direct route regardless of her type. By using an indirect route, the investor bears the risk of paying the penalty tax but does not earn greater net-of-tax income than by using the direct route. Under such tax credit systems, treaty shopping can be prevented in equilibrium, as suggested by Theorem 4.

However, foreign tax credit systems do not always help prevent treaty shopping. Under tax credit systems with $t_1 < t_{s1}$, there exists an equilibrium where the investor chooses indirect routes with positive probability, as shown by Theorem 5. This equilibrium is similar to the equilibrium of Theorem 2 that holds under deduction or exemption systems.

Treaty shopping can be prevented by foreign tax credit systems in residence countries with relatively high tax rates. When no treaty shopping occurs, the source country earns the maximum possible tax revenue. However, this comes at the cost of tax credits given by the residence countries. The source country benefits from foreign tax credit systems in the residence
countries that pay for the cost of the systems. Without overcoming such free-rider problems between countries, it may be difficult to sustain foreign tax credit systems in the residence countries. While China, Korea, and the United States maintain foreign tax credit systems, France, Germany, Japan, and the United Kingdom have moved away from these systems to introduce exemption for foreign-source dividends.

5 Conclusion

In real-world cases of tax treaty shopping, investors own and operate conduit entities in countries with favorable treaties, and tax authorities may not know the true residency of the investors. To better understand the behavior of investors, or unknown beneficial owners, in this paper, I develop a game-theoretic model with incomplete information about the types of investors and tax authorities, and examine a Bayesian Nash equilibrium where an investor of a country with tax-minimizing indirect routes uses such indirect routes with positive probability. I also compute tax revenue loss due to the use of indirect routes for treaty shopping, and show the payoff dominance of a random audit strategy to the pure always-audit strategy. Under foreign tax credit systems in residence countries with relatively high tax rates, there is an equilibrium where an investor chooses a direct route regardless of her type, and thus, treaty shopping can be prevented in equilibrium.

For future studies it will be important to examine foreign tax credit systems from the perspectives of capital exporting or investors’ residence countries. Foreign tax credit systems can help prevent treaty shopping but may also be vulnerable to other tax planning techniques, such as “foreign tax credit generators,” unfairly reducing tax revenue in residence countries. In an extended model, residence countries may choose tax relief systems to maximize their own revenue, and then investors and source countries may play the game of treaty shopping and tax auditing, as examined in this paper.

It will also be interesting to study a model where an investor can choose a destination of investment. In this paper, given a pair of residence and source
countries, an investor chooses an investment route, direct or indirect, from the residence to the source (destination) country. However, if the tax agency of the source country is expected to be aggressive, or to audit frequently, the investor may want to adjust her destination. Moreover, expecting this adjustment, the tax agency may have to be more accommodating toward the investor who uses a tax-minimizing indirect route.

I wish to address these issues in my future research.

Appendix

Proof of Theorem 4. This proof proceeds in three steps.

Step 1. For the investor of type \((1, m)\), it is weakly dominant to choose the direct route. For each \(m \in M\), let \(b^*(1, m) = 1\), \(s\) be the direct route. For each \(a(c) \in \{0, 1\}\), \(p(a(c), b^*(1, m)) = 0\). Because \(f(b^*(1, m)) = t_{s1}\) and \(t_{s1} \leq t_1\), \(u_B(a(c), b^*(1, m); 1, m) = m(1 - \max\{t_{s1}, t_1\}) = m(1 - t_1)\). For each \(b \in R(1)\) with \(b \neq b^*(1, m)\), if \(a(c) = 0\), then \(p(a(c), b) = 0\) and \(u_B(a(c), b; 1, m) = m(1 - \max\{f(b), t_1\})\). Because \(t_1 \leq \max\{f(b), t_1\}\), \(u_B(a(c), b; 1, m) \leq u_B(a(c), b^*(1, m); 1, m)\). Also, if \(a(c) = 1\), \(p(a(c), b) = p^*\) and \(u_B(a(c), b; 1, m) = m(1 - \max\{f(b), t_1\}) - E_\phi [mp^*]\). Because \(E_\phi [mp^*] > 0\) and \(t_1 \leq \max\{f(b), t_1\}\), \(u_B(a(c), b; 1, m) < u_B(a(c), b^*(1, m); 1, m)\).

Step 2. For the investor of type \((2, m)\), it is weakly dominant to choose the direct route. For each \(m \in M\), let \(b^*(2, m) = 2\), \(s\) be the direct route. Because country 2 belongs to \(H^D\), the direct route is tax-minimizing, i.e., for each \(b \in R(2)\) with \(b \neq b^*(2, m)\), if \(a(c) = 0\), then \(p(a(c), b^*(2, m)) = p(a(c), b) = 0\). Thus, \(u_B(a(c), b; 2, m) = m(1 - \max\{f(b), t_2\})\) and \(u_B(a(c), b^*(2, m); 2, m) = m(1 - \max\{t_{s2}, t_2\})\) since \(f(b^*(2, m)) = t_{s2}\). Because \(t_{s2} \leq f(b)\) implies \(\max\{t_{s2}, t_2\} \leq \max\{f(b), t_2\}\), \(u_B(a(c), b; 2, m) \leq u_B(a(c), b^*(2, m); 2, m)\). Also, for each \(b \in R(2)\) with \(b \neq b^*(2, m)\), if \(a(c) = 1\), then \(p(a(c), b^*(2, m)) = 0 < p^* = p(a(c), b)\). Thus, \(u_B(a(c), b; 2, m) = m(1 - \max\{f(b), t_2\}) - E_\phi [mp^*]\), and since \(f(b^*(2, m)) = t_{s2}\), \(u_B(a(c), b^*(2, m); 2, m) = m(1 - \max\{t_{s2}, t_2\})\). Because \(E_\phi [mp^*] > 0\),
similarly as above, \( u_B(a(c), b; 2, m) < u_B(a(c), b^*(2, m); 2, m) \).

**Step 3.** From the previous steps, for the investor of both types, it is weakly dominant to choose the direct route. If the investor chooses the direct route regardless of her type, there is no penalty tax even if the tax agency audits the investor. Thus, to maximize the payoff, the tax agency chooses no audit regardless of its type. Hence, there is an equilibrium where the tax agency chooses no audit and the investor chooses the direct route regardless of their types. □

**Proof of Theorem 5.** This proof proceeds in five steps.

**Step 1.** For the tax agency of type \( c_H \), it is dominant to choose not to audit. Let \( a^*(c_H) = 0 \) and \( a(c_H) = 1 \). For each strategy \( b(\cdot) \) of the investor, \( u_A(a^*(c_H), b(\cdot); c_H) = E_\pi[mp(b(\cdot))] \). Also, \( u_A(a(c_H), b(\cdot); c_H) = E_\pi[mp(c_H)] - c_H \). Because \( E_\pi[mp(a(c_H), b(\cdot))] \leq p^*E_\pi[m] < c_H \), it holds that \( u_A(a(c_H), b(\cdot); c_H) < u_A(a^*(c_H), b(\cdot); c_H) \).

**Step 2.** For the investor of type \((2, m)\), it is weakly dominant to choose the direct route. For each \( m \in M \), let \( b^*(2, m) = 2, s \) be the direct route. Because country 2 belongs to \( H^D \), the direct route is tax-minimizing, i.e., for each \( b \in R(2) \), \( f(b^*(2, m)) \leq f(b) \). For each \( b \in R(2) \) with \( b \neq b^*(2, m) \), if \( a(c) = 0 \), then \( p(a(c), b^*(2, m)) = p(a(c), b) = 0 \). Thus, \( u_B(a(c); b; 2, m) = m(1 - \max\{f(b), t_2\}) \) and \( u_B(a(c), b^*(2, m); 2, m) = m(1 - \max\{t_{s2}, t_2\}) \) since \( f(b^*(2, m)) = t_{s2} \). Because \( t_{s2} \leq f(b) \) implies \( \max\{t_{s2}, t_2\} \leq \max\{f(b), t_2\} \), \( u_B(a(c), b; 2, m) \) \leq \( u_B(a(c), b^*(2, m); 2, m) \). Also, for each \( b \in R(2) \) with \( b \neq b^*(2, m) \), if \( a(c) = 1 \), then \( p(a(c), b^*(2, m)) = 0 < p^* = p(a(c), b) \). Thus, \( u_B(a(c), b; 2, m) = m(1 - \max\{f(b), t_2\}) - E_\phi[mp^*] \), and since \( f(b^*(2, m)) = t_{s2} \), \( u_B(a(c), b^*(2, m); 2, m) = m(1 - \max\{t_{s2}, t_2\}) \). Because \( E_\phi[mp^*] > 0 \), similarly as above, \( u_B(a(c), b; 2, m) < u_B(a(c), b^*(2, m); 2, m) \).

**Step 3.** For the investor of type \((1, m)\), it is weakly dominated to choose an indirect route that is not tax-minimizing. For each \( m \in M \), let \( b(1, m) = b \) denote such a route. Since country 1 belongs to \( H^I \), there is a tax-minimizing indirect route \( b^* = 1, i, \ldots, j, s \) such that \( f(b^* < t_{s1} \). Let \( b^*(1, m) = b^* \).
Because $b$ and $b^*$ are indirect routes, for each $a(c) \in \{0,1\}$, $p(a(c), b) = p(a(c), b^*)$. Because $b^*$ is tax-minimizing but $b$ is not, $f(b^*) < f(b)$, which implies $\max\{f(b^*), t_1\} \leq \max\{f(b), t_1\}$. Thus, for each $a(c) \in \{0,1\}$ and each $m \in M$, it holds that $u_B(a(c), b; 1, m) \leq u_B(a(c), b^*; 1, m)$.

**Step 4.** Suppose that there are $\ell \geq 1$ tax-minimizing indirect routes from country 1 to country $s$. Each of such routes is denoted by $b^k = 1, i_k, \ldots, j_k, s$ with $k = 1, \ldots, \ell$, and is played with probability $q_k$, where $\sum_{k=1}^{\ell} q_k = c_L/\bar{m}p^*$. Let $t_f = \max\{f(b^*), t_1\}$. Let $(a^*(\cdot), b^*(\cdot))$ denote the strategy profile specified as follows:

$$a^*(c) = \begin{cases} 1 \text{ with probability } (t_{s1} - t_f)/\phi_Lp^* \text{ for } c = c_L \\ 0 \text{ with probability } 1 - (t_{s1} - t_f)/\phi_Lp^* \text{ for } c = c_L \\ 0 \text{ with probability } 1 \text{ for } c = c_H \end{cases}$$

$$b^*(h, m) = \begin{cases} b^k \text{ with probability } q_k \text{ for } h = 1 \\ 1, s \text{ with probability } 1 - \sum_{k=1}^{\ell} q_k \text{ for } h = 1 \\ 2, s \text{ with probability } 1 \text{ for } h = 2 \end{cases}$$

Given the investor’s strategy $b^*(\cdot)$, it holds that $E_\pi [\bar{m}p(1, b^*(\cdot))] = c_L$ and $u_A(1, b^*(\cdot); c_L) = E_\pi [\bar{m}w(b^*(\cdot))]$. Also, $u_A(0, b^*(\cdot); c_L) = E_\pi [\bar{m}w(b^*(\cdot))]$. Thus, the tax agency of type $c_L$ is indifferent between $a^*(c_L) = 1$ and $a^*(c_L) = 0$. Given the tax agency’s strategy $a^*(\cdot)$, for each $m \in M$, it holds that $u_B(a^*(\cdot), b^k; 1, m) = m(1 - t_{s1}) = u_B(a^*(\cdot), 1, s; 1, m)$. Thus, the investor of type $(1, m)$ is indifferent between $b^*(1, m) = b^k$ and $b^*(1, m) = 1, s$.

**Step 5.** From Step 1, it is dominant for the tax agency of type $c_H$ to choose no audit. Regardless of what the investor does, the tax agency of type $c_H$ will choose no audit, as specified in $a^*(\cdot)$. From Step 2, it is weakly dominant for the investor of type $(2, m)$ to choose the direct route. Thus, regardless of what the tax agency does, the investor of type $(2, m)$ will choose the direct route, as specified in $b^*(\cdot)$. Because it is weakly dominated for the investor of type $(1, m)$ to choose an indirect route that is not tax-minimizing, from Step 3, the investor of type $(1, m)$ will only choose either the direct route or a tax-minimizing indirect route. By choosing the mixed strategy specified in
Step 4, each player makes the other player indifferent between the actions played with positive probability. Hence, \((a^*(\cdot), b^*(\cdot))\) is an equilibrium. □

References


