재정학연구 제15권 제3호(통권 제114호) 37-64

On the Incompatibility between the Pareto Principle and the Separability Principle*

Inkee Jang**

Abstract We study egalitarianism and the logical relations among the principles of fairness on public financial distributions. It is known that the separability principle, which is a strand of fairness axioms, tends to be incompatible with the standard Pareto principle. We investigate the logical relations between the separability principle and the standard Pareto principle in the context of public finance. We adopt Internal Separability as the separability principle, adopt Permutation Pareto Principle as a restricted form of the standard Pareto principle, and study the incompatible relationship between the two axioms. We also provide an impossibility result that a social ordering cannot satisfy the two axioms along with a restricted form of continuity and an additional restricted weak standard Pareto principle, unless every agent's preference is identical. Finally, we relieve the incompatibility result by introducing Weak Internal Separability and showing that it is implied by Internal Separability as well as it is compatible with Permutation Pareto Principle.

Key Words: **Standard Pareto Principle, The Separability Principle, Egalitarianism, Internal Separability, Permutation Pareto Principle**

JEL Code: D63, D71

Received: March 28, 2022. Revised: April 11, 2022. Accepted: Aug. 11, 2022.

* I thank Jonathan Weinstein, Paulo Natenzon, and Kee-Youn Kang for their contributions and advice to this paper. No potential competing interest was reported by the authors.

** Assistant Professor, Department of Economics, The Catholic University of Korea, 43 Jibong-ro, Bucheon-si, Gyeonggi-do 14662, Republic of Korea, e-mail: inkeejang@catholic.ac.kr

Ⅰ. Introduction

Achieving fairness in public financial subsidy or distribution to individuals is considered a common task for all social systems. The fairness of financial distribution protects members of society in spots from the risk of fatal financial losses that can come to anyone. However, members of society are in different situations and, more importantly, have different attributes. Even though society makes several decisions on behalf of members of society, society's egalitarian choices are generally not supported by all members due to their diversity.

In the respect of the diversity of individual preferences, it can be assumed that society has a preference, independent of individual preferences, even if its choices are not always supported by all members. In the health sector, for example, the pursuit of fairness in the distribution of social resources is often taken for granted, while the direction and extent of fairness are bound to involve the value judgment of the society. As a practical instance, given the findings that medical expenses for severe smokers, overweight people, and alcoholics are higher (Leigh and Fries, 1992; Viscusi and Hersch, 2008), it can be controversial whether society should financially subsidize the health consequences of dangerous behaviors. For another example, the problem of fair allocation can be considered in the context that individuals have different capabilities for transforming resources into basic human functionings (Moreno-Ternero and Roemer, 2006; Chun et al., 2014). An important view of fairness in this context is Rawls' difference principle, which insists that it is just to make the least advantaged in society materially better off. For the abstraction of these various contexts, this paper, as well as many studies, regards each individual's attributes or opinions as individual preferences, assumes that society also has a preference, and discusses the egalitarian properties that are desirable for the social preferences to have.

This paper aims to contribute to the discussion on egalitarian social preference, especially in the multi-commodity context of social ordering. In general, egalitarianism does not necessarily mean distributing all goods equally to all individuals. We regard egalitarianism as a political philosophical position that puts the value of equality first and that, like other related studies, different distributions among individuals are acceptable and desirable only when it is based on differences in their preferences. Based on this idea, it is argued that the difference in distribution should be based on the utility of agents, not on other attributes or political interests of agents.

A vast amount of literature studied egalitarianism in the theory of public finance, and it began in the single commodity context. Hardy et al. (1934) showed that Pigou-Dalton (Pigou, 1912; Dalton, 1920) transfer dominance and Lorenz dominance are equivalent. Atkinson (1970) studied the implications and limitations of the Lorenz dominance. Sen (1973) introduced the weak equity axiom as an egalitarian axiom which stipulates that income should be distributed so that those with a low general capacity to achieve welfare receive a higher income than those with high general capacity. Hammond (1976) introduced the principle of equity (also called Hammond equity axiom) in the context of social choice, and in the spirit of Sen's weak equity axiom. There are several variations of the Hammond equity axiom (see Asheim et al., 2007; Banerjee, 2006), including Strong equity (see d'Aspremont and Gervers, 1977; Dubey and Mitra, 2014).

On the other hand, most of the recent research on egalitarian social ordering has focused on multi-dimensional environments since Kolm (1977) first showed the dominance principle on multi-dimensional egalitarianism. Koshevoy (1995), Koshevoy and Mosler (1996), and Koshevoy (1997) deepened the work of Kolm by studying the generalization of Lorenz dominance in a multi-dimensional commodity space. Muller and Trannoy (2012) applied the dissymmetric preference and endowment of individuals in a multi-dimensional setting. However, Marshall et al. (1979) showed that the results of one-dimensional studies are difficult to be extended to multi-dimensional public finance models.

A difficulty of the multi-dimensional public finance model other than the extension from the single-dimensional studies is the incompatible relationship between transfer principles, one strand of the representative equity axioms, and the standard Pareto principle. The standard Pareto principle says that for any multi-commodity allocations $x=(x_1,x_2,\ldots,x_n)$ and $y=(y_1,y_2,\ldots,y_n)$, where x_i denotes the bundle for agent i in the allocation x and y_i denotes the bundle for agent i in the allocation y, if every agent prefers its bundle in y to that in x, then the society should also prefer y to x. Moyes (1999) introduced the transfer principles as the basis of dominance criteria. Fleurbaey and Trannoy (2003) showed that, in a multi-commodity model where diversity of individual preferences is allowed, even the weak standard Pareto principle and the Dominance-Reducing Transfer principle are incompatible. The Dominance-Reducing Transfer principle argues that for any allocation $x=(x_1,x_2,\ldots,x_n)$, any transfer that reduces a dominance between any two bundles is justifiable. Specifically, if a transfer t satisfies $x_i < x_i + t \leq x_j - t < x_j$ for any $i,j \in \{1,\ldots,n\}$, then society should prefer $(x_1,\ldots,x_{i-1},x_i + t,x_{i+1},\ldots,X_{j-1},x_j - t,x_{j+1},\ldots,x_n)$ to x. Fleurbaey (2005), Fleurbaey (2007), and Fleurbaey and Maniquet (2008) introduced weaker axioms of the Dominance-Reducing Transfer principle to be compatible with the standard Pareto principle. On the other hand, Sprumont (2012) showed that a weaker axiom than the standard Pareto principle, which is called Consensus, is compatible with a stronger axiom than the Dominance-Reducing Transfer principle, which is called Dominance Aversion. Jang (2017) extended the work of Sprumont by showing that even a stronger axiom than Dominance Aversion is compatible with Permutation Pareto Principle, an alternative weaker axiom than the standard Pareto principle, along with Consensus. Specifically, Jang (2017) introduced and applied Internal Dominance,[1] a separability principle, and the Permutation Pareto Principle to the social ordering he axiomatized. Internal Dominance contains a partial idea of a lexicographic preference as well as an imperfect separability property. Permutation Pareto Principle says that if permuting bundles can result in every agent preferring her new bundle to the old, then this new allocation should be considered better.

A strand of the representative equity axiom other than the transfer principle is the separability principle. However, while much of the previous research has focused on logical relations between the Dominance-Reducing Transfer principle along with the standard Pareto principle, relatively less is studied about the incompatible relationship between the separability principle and the standard Pareto principle. The separability principle requires that if some of the agents' preferences are subject to change, and if there is a subgroup of agents with the same preferences and the total amount of public funds granted to this subgroup is the same, the amount assigned to each agent in the subgroup should be the same. Moulin (1987) first introduced the principle in the context of surplus sharing, and Chun (1999) and Chun (2000) contributed to the study of the separability principle in the context of bankruptcy problem and quasi-linear social choice theory. Separability can also be thought of as relative to the additivity[2] and consistency axioms,[3]

---

1) Internal Dominance states that if the society prefers allocation x over allocation y, and if x_i and y_i are considered indifferent by the society, then there are bundles x_i',y_i' such that when these bundles replace x_i and y_i respectively, any permutation of $(x_i'; x_{-i})$ is strictly preferred to $(y_i'; y_{-i})$ by the society.

which are two of the leading axioms in the public finance theory. The standard leximin social orderings are subgroup separable, and both Sprumont (2012) and Jang (2017) provide an axiomatization of a family of leximin social orderings.

This article is also related to the literature on the results of impossibility in social choice theory. We focus on the incompatibility between the separability principle and the standard Pareto principle. Since the seminal and well-known impossibility theorem, e.g., Condorcet's voting paradox and Arrow's impossibility theorem, there has been a vast literature that contributes to the impossibility research. Since Arrow (1950) showed an epoch-making impossibility result for the social welfare function, many papers applied Arrow's theorem beyond his original context. See Dietrich and List (2007), Leitgeb and Segetberg (2007), Okasha (2011), and Stegenga (2013) for examples. Diamond (1965) introduced the celebrated theorem to show that there is no social welfare function that satisfies the standard Pareto principle, intergenerational equity, and continuity. Svensson (1980), on the other hand, established the general possibility result showing that there exist orderings that satisfy the standard Pareto principle and the intergenerational equity. Basu and Mitra (2003) enriched the work of Diamond (1965) and Svensson (1980) by showing that there is no social welfare function that satisfies the standard Pareto principle and the equity axioms.

We provide an impossibility result on a restricted standard Pareto and an equity axiom in social ordering. To be specific, we provide an incompatibility result between Internal Separability and Permutation Pareto Principle. Internal

---

2) Shapley (1953) used the additivity axiom to provide a classic axiomatization result and Moulin (2013) provided an application discussion on additivity. Also see Moulin (1987) and Ju et al. (2007).

3) Hart and Mas-Collel (1989) provided an axiomatization invoking consistency.

Separability is a separability axiom arguing that when the society deems a bundle a as valuable as a bundle a' and a bundle b as valuable as a bundle b', a certain agent's preference can be ignored. That is, for a social ordering $\boldsymbol{R}$, if $(a,...,a)\boldsymbol{I}(a',...,a')$ and $(b,...,b)\boldsymbol{I}(b',...,b')$, where $\boldsymbol{I}$ denotes the indifference relation of the social ordering $\boldsymbol{R}$, then for all allocations x,x' and any agent i, $(a;x_{-i})\boldsymbol{R}(a';x'_{-i})$ if and only if $(b;x_{-i})\boldsymbol{R}(b';x'_{-i})$. We also introduce an axiom, Indifference Unanimity, that helps further the main discussion. Indifference Unanimity is a demanding property requiring that the society can evaluate two bundles to be indifferent only when all the individuals agree that the two bundles are indifferent. That is, for a social ordering $\boldsymbol{R}$ and any two bundles a and b, $(a,...,a)\boldsymbol{I}(b,...,b)$ only when a and b are indifferent toall agents. We eventually show an impossibility result that, given that individual preferences are diverse, there does not exist a social ordering that satisfies Consensus, Internal Separability, Permutation Pareto Principle, and a restricted continuity property.

We also introduce an axiom, Weak Internal Separability, which is a weaker axiom thanInternal Separability and is compatible with Permutation Pareto Principle. Weak Internal Separability argues that, unlike Internal Separability, even with $(a,...,a)\boldsymbol{I}(a',...,a')$ and $(b,...,b)\boldsymbol{I}(b',...,b')$, it is allowed to ignore an individual i's preference only when the i's ranking between a and b and between a' and b' are identical. The introduction of this axiom is meaningful in that it indirectly shows that the concepts of Internal Separability and Permutation Pareto Principle are not antagonistic, even though they are logically incompatible with each other. Jang (2017) also introduced an axiom, Weak Internal Dominance, that is implied by Internal Separability and is compatible with Permutation Pareto Principle. However, Weak Internal Dominance ignores individual preferences, as same as Internal Separability, and it is too weak an axiom. We prove that, in an environment where individuals' preferences are diverse, Weak Internal Separability is a stronger axiom than Weak Internal Dominance, to show the significance of Weak Internal Separability.

This paper is organized as follows. Section 2 introduces the model and conditions. Section 3 spreads the axioms and the main results. Section 4 concludes. We provide proofs of propositions in Appendix A.

Ⅱ. Preliminaries

There is a fixed finite number of commodities $m \geq 2$ in an economy and a fixed finite set of agents $N = \{1, \ldots, n\}$ such that $n \geq 2$. Let $X = \mathbb{R}_+^m$ be the commodity space, or the set of bundles, and let X^N be the set of conceivable allocations. Each agent $i \in N$ has a continuous, rational, and strictly monotonic (preference) ordering R_i over X, while each social ordering $\boldsymbol{R}$ is rational and is over X^N. P_i denotes the strict preference relation associated with R_i, and $\boldsymbol{P}$ denotes the strict preference relation associated with $\boldsymbol{R}$. For any allocation $x \in X^N$ and any permutation $\pi : N \rightarrow N$, let $\pi(x)$ denote $(x_{\pi(1)}, \ldots, x_{\pi(n)})$. Also, for any $x, y \in X^N$ and any permutation π, we say that y is permuted from x with π when $y = \pi(x)$. For any k-dimensional vectors a and b, $a \geq b$ if and only if $a_i \geq b_i$ for all $i \in \{1, \ldots, k\}$, $a > b$ if and only if $a_i \geq b_i$ for all $i \in \{1, \ldots, k\}$ and the inequality is strict for at least one case. For any $x = (x_1, \ldots, x_n) \in X^N$ and any preference relation R, let $x^R = (x_1^R, \ldots, X_n^R)$ denote a rearrangement of the bundles in x from the worst to the best according to R, that is, $x_n^R R x_{n-1}^R R \ldots R x_1^R$ with a tie-breaking rule: if $x_i I x_j$ for any $i, j \in N$ such that $i < j$, x_i is arranged before x_j in x^R. For any $x, y \in X^N$, $y \geq_{par} x$ denotes that y Pareto improves x, namely, $y_i R_i x_i$ for all $i \in N$, and $y >_{par} x$ denotes that y strictly Pareto improves x, namely, $y_i R_i x_i$ for all $i \in N$ with $y_j P_j x_j$ for at least one $j \in N$.

Even though social ordering is a preference relation over allocations, there is an indirect way for a social ordering to express a preference over bundles. For any two bundles $a, b \in X$, we can interpret that a social ordering $\boldsymbol{R}$ prefers b to a if $\boldsymbol{R}$ prefers a fully egalitarian allocation where all the bundles are b to a fully egalitarian allocation where all the bundles are a, namely, $(b, \ldots, b) \boldsymbol{R} (a, \ldots, a)$.

Ⅲ. Main Results

This section studies the logical relations between the separability principle and the Pareto principle, provides an impossibility result, and introduces an axiom that mitigates the incompatible relation between two axioms.

1. Internal Separability and Permutation Pareto Principle

The axioms we study in this section are closely related to the family of lexicographic social orderings,[4] which has a well-known problem of discontinuity. We apply a limited form of continuity axiom, Weak Continuity, throughout the discussion. Weak Continuity requires the social ordering to be continuous while it compares bundles. In other words, Weak Continuity argues that the social ordering should be continuous for fully egalitarian allocations.

4) Both Sprumont (2012) and Jang (2017) characterized a family of leximin social orderings.

Weak Continuity. For any $a,b \in X$, and any sequence $\{b^k\}$ in X converging to b, $(b^k,\ldots,b^k)\boldsymbol{R}(a,\ldots,a)$ for all k implies $(b,\ldots,b)\boldsymbol{R}(a,\ldots,a)$.

It is reasonable to invoke Weak Continuity in the environment where agents' preferences are continuous. In other words, because all the agents in the society compare bundles in a continuous manner, so should the society. For that reason, in fact, in Sprumont (2012) and Jang (2017), Weak Continuity is laid as a basic axiom for axiomatization.

To relieve the incompatibility result between the standard Pareto principle and the Dominance-Reducing Transfer principle, Sprumont (2012) introduced Consensus, a weaker replacement of the standard Pareto principle, and Internal Separability, a separability principle. Consensus says that an allocation y is preferred to an allocation x if all the individuals prefer every bundle y_i in the former allocation to every bundle x_i in the latter.

Consensus. For any $x,y \in X^N$, if $y_i P_j x_i$ for all $i,j \in N$, then $y\boldsymbol{P}x$.

Internal Separability (Sprumont, 2012) says that when the society considers a bundle a as valuable as a bundle a', and a bundle b as valuable as a bundle b', any single agent i's preference can be ignored.

Internal Separability. Let $i \in N$, $x,x' \in X^N$, and $a,a' \in X$ such that $(x_i,\ldots,x_i)\boldsymbol{I}(x_i',\ldots,x_i')$ and $(a,\ldots,a)\boldsymbol{I}(a',\ldots,a')$. Then, $x\boldsymbol{R}x'$ if and only if $(a;x_{-i})\boldsymbol{R}(a';x'_{-i})$.

Internal Separability is a separability principle in that the comparison of two allocations x and x' is equivalent to the comparison of x_{-i} and x'_{-i}, while i's bundle can be well-substituted by a and a'.

Sprumont (2012) introduced an axiom, Dominance Aversion, which is stronger than the Dominance-Reducing Transfer principle, and showed that there exists a social ordering that satisfies Consensus, Internal Separability, and Dominance Aversion. Dominance Aversion says that any change that reduces any bundle dominance is always desirable.[5] Notably, Consensus is a weak enough Paretian axiom to be compatible with the Dominance-Reducing Transfer principle and with the separability principle, unlike the standard Pareto principle.

Jang (2017) enriched the work on the incompatibility issue between the standard Pareto principle and the separability principle. He showed an axiomatization result of a social ordering that satisfies a stronger axiom than Dominance Aversion, named Strong Dominance Aversion, and also satisfies not only Consensus but also another Paretian axiom named Permutation Pareto Principle. Strong Dominance Aversion argues that reducing bundle dominance and maintaining the egalitarian order is desirable. That is, for any $x,y \in X^N$ and any $i,j \in N$, if $x_i > y_i$, $(y_i,\ldots,y_i)\boldsymbol{R}(y_j,\ldots,y_j)$, and $y_j > x_j$ and $y_k = x_k$ for all $k \in N \setminus \{i,j\}$, then $y\boldsymbol{R}x$.[6]

Permutation Pareto Principle is a weaker replacement of the standard Pareto principle, which says that if an allocation can be Pareto improved by being permuted, then the permuted allocation should be preferred to the original

5) Dominance Aversion argues that for any $x,y \in X^N$ and any $i,j \in N$, if $x_i > y_i \geq y_j > x_j$ and $y_k = x_k$ for all $k \in N \setminus \{i,j\}$, then $y\boldsymbol{R}x$. It trivially implies the Dominance-Reducing Transfer principle.

6) Strong Dominance Aversion implies Dominance Aversion if social orderings are assumed to be monotonic. Even without the monotonicity of social orderings, Strong Dominance Aversion implies Dominance Aversion if Weak Continuity and Consensus are satisfied. In other words, any social ordering that satisfies Strong Dominance Aversion, Weak Continuity, and Consensus also satisfies Dominance Aversion. To be specific, Strong Dominance Aversion implies Dominance Aversion if $b \geq a$ implies $(b,\ldots,b)\boldsymbol{R}(a,\ldots,a)$ for any bundles $a,b \in X$. Note that, by Consensus and the fact that every agent's preference is strictly monotonic, $b > a$ implies $(b,\ldots,b)\boldsymbol{P}(a,\ldots,a)$, Finally, by Weak Continuity, $b \geq a$ implies $(b,\ldots,b)\boldsymbol{R}(a,\ldots,a)$.

allocation.

Permutation Pareto Principle. For any $x \in X^N$ and any permutation π on N, $\pi(x) \geq_{par} x$ implies $\pi(x)\boldsymbol{R}x$ and $\pi(x) >_{par} x$ implies $\pi(x)\boldsymbol{P}x$.

Even though the Permutation Pareto Principle is weaker enough than the standard Pareto principle so that it is compatible with Strong Dominance Aversion, Permutation Pareto Principle still has an incompatibility problem with Internal Separability, unlike Consensus.

We introducea new axiom, Indifference Unanimity, to deepen and further the discussion on the incompatibility result. Indifference Unanimity requires that society can evaluate two bundles to be indifferent only when all the individuals agree that the two bundles are indifferent. In other words, if at least one of the individuals judges that some two bundles are not indifferent, then the social planner cannot deem one bundle as equally valuable as the other.

Indifference Unanimity. For any $a,b \in X$, $(b,\ldots,b)\boldsymbol{I}(a,\ldots,a)$ implies $bI_i a$ for all $i \in N$.

Indifference Unanimity is a demanding property; it requires a strict standard for allocations to be evaluated as indifferent by a social ordering. It may be argued that it is not unreasonable for a society to be indifferent between allocations $(a,\ldots,a)$ and $(b,\ldots,b)$ when one agent prefers a to b, while another agent prefers b to a.

It turns out that the combination of Consensus, Internal Separability, and Permutation Pareto Principle implies Indifference Unanimity. To provide an intuition, consider a society in which there are two agents and where the bundle space is two-dimensional, and let any social ordering $\boldsymbol{R}$ that satisfies Consensus, Internal Separability, and Permutation Pareto Principle. Note that Indifference Unanimity says that, for any bundles a and b, if "$bI_i a$ for all $i \in N$" does not hold, then $(b,b)\boldsymbol{I}(a,a)$ also does not hold. To argue that $\boldsymbol{R}$ satisfies Indifference Unanimity, without loss of generality, suppose that $aP_1 b$ and $bR_2 a$. Then $\boldsymbol{R}$ satisfies Indifference Unanimity if and only if $(b,b)\boldsymbol{I}(a,a)$ does not hold. Suppose conversely that $(b,b)\boldsymbol{I}(a,a)$ holds. Then, because $\boldsymbol{R}$ satisfies Internal Separability, we can apply $(b,b)\boldsymbol{I}(a,a)$ to $(a,a)\boldsymbol{I}(a,a)$ to get $(a,b)\boldsymbol{I}(a,a)$ and $(b,a)\boldsymbol{I}(a,a)$, which in turn implies $(a,b)\boldsymbol{I}(b,a)$. However, $aP_1 b$ and $bR_2 a$, along with Permutation Pareto Principle, implies $(a,b)\boldsymbol{P}(b,a)$, which leads to a contradiction. The following proposition formally states this result: Internal Separability and Permutation Pareto Principle, along with Consensus, imply Indifference Unanimity. This logical relation implies that applying Internal Separability and Permutation Pareto Principle to a social ordering puts a severe restriction on it.

Proposition 1. *Consensus, Internal Separability, and Permutation Pareto Principle imply Indifference Unanimity.*
Proof. See Appendix A.

The restriction that Internal Separability and Permutation Pareto Principle place on the social ordering's indifference evaluation prevents social order from satisfying Weak Continuity. To be specific, consider three bundles $a,b,c \in X$ such that $(b,\ldots,b)\boldsymbol{P}(a,\ldots,a)$ and $(a+c,\ldots,a+c)\boldsymbol{P}(b,\ldots,b)$. Note that $c>0$ is necessary. Weak Continuity requires that there should be some $t \in (0,1)$ such that $(b+t\cdot c,\ldots,b+t\cdot c)\boldsymbol{I}(a,\ldots,a)$. At the same time, however, it is unlikely that $(b+t\cdot c)I_i a$ for all $i \in N$ hold, given that a,b,c are arbitrary enough. That is, it is likely for Indifference Unanimity to insist that

$(b+t\cdot c,\ldots,b+t\cdot c)\boldsymbol{I}(a,\ldots,a)$ cannot hold. However, this conflict does not occur if every agent's preference is identical. Specifically, if we assume that every agent's preference is identical, it is reasonable to argue that $(b,\ldots,b)\boldsymbol{P}(a,\ldots,a)$ and $(a+c,\ldots,a+c)\boldsymbol{P}(b,\ldots,b)$ if and only if $bP_i a$ and $(a+c)P_i b$ for all $i\in N$. Moreover, because individual preferences are continuous, there exists $t\in(0,1)$ such that $(b+t\cdot c)I_i a$ for all $i\in N$, which can indicate $(b+t\cdot c,\ldots,b+t\cdot c)\boldsymbol{I}(a,\ldots,a)$ when every agent has the same preference.

The following proposition formally states this result: unless we put a strict assumption that every agent has the same preference, Internal Separability and Permutation Pareto Principle cannot satisfy together, along with Consensus and Weak Continuity.

Proposition 2. *There is no social ordering satisfying Consensus, Weak Continuity, Internal Separability, and Permutation Pareto Principle unless* $R_i = R_j$ *for all* $i,j\in N$.
Proof. See Appendix A.

Proposition 1 and 2eventually state that it is impossible to combine Consensus, Weak Continuity, Internal Separability, and Permutation Pareto Principle, whenever the agents' preferences are diverse.

2. Weak Internal Separability

Permutation Pareto Principle fails to be compatible with Internal Separability, even though it requires less enough than the standard Pareto principle. It is, thus, reasonable to interpret that Internal Separability is a strong separability principle that is difficult to be together with Paretian axioms. For that reason, we introduce Weak Internal Separability, which is weaker than Internal Separability and is compatible with Permutation Pareto Principle.

Jang (2017)already proposed an axiom, Weak Internal Dominance, that is implied by Internal Separability and showed that it is compatible with Permutation Pareto Principle. Weak Internal Dominance says that when the society considers a bundle a as valuable as a bundle a', then for any bundle b' and any agent i, there exists a bundle b that maintains the social ranking of allocations x and x', in other words, enables to ignore the preference of agent i.

Weak Internal Dominance. For any $a' \in X$ and $x, x' \in X^N$ such that $(x_i, \ldots, x_i) \boldsymbol{I} (x_i', \ldots, x_i')$ for some $i \in N$, there exists $a \in X$ such that $x \boldsymbol{R} x'$ implies $(a; x_{-i}) \boldsymbol{R} (a'; x'_{-i})$.

It is trivial that Internal Separability implies Weak Internal Dominance.[7] Weak Internal Dominance requires less than Internal Separability in two different channels: *i)* it does not require that the ranking of x and x' and the ranking of $(a; x_{-i})$ and $(a'; x'_{-i})$to be identical, and *ii)* $(a, \ldots, a) \boldsymbol{I} (a', \ldots, a')$ is not required to allow disregard of agent i's preference.

We criticize Weak Internal Dominance to explain the significance of introducing Weak Internal Separability. First, Weak Internal Dominance lacks consideration for individual preferences. Notice that Internal Separability has room for criticism that ignores a certain agent i's preference using the social

7) Consider any social ordering $\boldsymbol{R}$ that satisfies Internal Separability. Also consider any $a' \in X$, and $x, x' \in X^N$ such that $(x_i, \ldots, x_i) \boldsymbol{I} (x_i', \ldots, x_i')$ for some $i \in N$. It is trivial that $(a, \ldots, a) \boldsymbol{I} (a', \ldots, a')$ with $a = a'$, thus, Internal Separability argues that $x \boldsymbol{R} x'$ implies $(a; x_{\{-i\}-i}) \boldsymbol{R} (a'; x'_{-i})$.

ordering's indifference relation as a tool, even though agent i may have a strict preference between x_i and x_i', or between a and a'. This characteristic is strongly related to the incompatible relation with the Permutation Pareto Principle. Weak Internal Dominance cannot avoid the same criticism even though it is compatible with Permutation Pareto Principle. It can be interpreted that the reason why Weak Internal Dominance is compatible with Permutation Pareto Principle despite its characteristic of ignoring preference is that Weak Internal Dominance is too weak an axiom. It is an axiom that is insufficient to be used to play an important role.

To prevent such unreasonable disregard of individual preferences and to make up for the weakness of Weak Internal Dominance, we argue that even if $(x_i,\dots,x_i)\boldsymbol{I}(x_i',\dots,x_i')$ and $(a,\dots,a)\boldsymbol{I}(a',\dots,a')$ are satisfied, agent i's preference is allowed to be ignored only when i's preference relations between x_i and x_i' and between a and a' are identical. Weak Internal Separability captures this idea.

Weak Internal Separability. Let $i\in N$, $x,x'\in X^N$, and $a,a'\in X$ such that $(x_i,\dots,x_i)\boldsymbol{I}(x_i',\dots,x_i')$ and $(a,\dots,a)\boldsymbol{I}(a',\dots,a')$. If either $[x_iP_ix_i'$ and $aP_ia']$, $[x_iI_ix_i'$ and $aI_ia']$, or $[x_i'P_ix_i$ and $a'P_ia]$, then $x\boldsymbol{R}x'$ if and only if $(a;x_{-i})\boldsymbol{R}(a';x'_{-i})$.

Notice that, unlike Internal Separability and Weak Internal Dominance, Weak Internal Separability requests a certain level of consideration of individual preference. Requiring a match in agent i's preference relations between the two pairs of bundles, $\{x_i,x_i'\}$ and $\{a,a'\}$, can be interpreted as seeking minimum consent from agent i. In this regard, because Weak Internal Separability has an additional requirement than Internal Separability to disregard agent i's preference, Internal Separability trivially implies Weak

Internal Separability. Weak Internal Separability, however, does not directly imply Weak Internal Dominance. Nevertheless, we argue that Weak Internal Separability is a more demanding axiom than Weak Internal Dominance by showing that Weak Internal Separability implies Weak Internal Dominance in a restricted environment. To specify the restriction on the environment, we state the following assumption.

Assumption 1. For each i, if aP_ia' and $(a,\ldots,a)\boldsymbol{I}(a',\ldots,a')$ for some $a,a'\in X$, then for any $b'\in X$, there exists $b\in X$ such that either bP_ib' or $b'P_ib$, and $(b,\ldots,b)\boldsymbol{I}(b',\ldots,b')$.

Although assumption 1 is strong, its implication is reasonable. aP_ia' and $(a,\ldots,a)\boldsymbol{I}(a',\ldots,a')$ mean that there is a pair of bundles $\{a,a'\}$ that the society values equally while agent i has a clear difference in preference. Assumption 1requires that the preference gap between the society and agent i is not limited to a specific pair $\{a,a'\}$. From a broader perspective, assumption 1 argues that various preferences of individuals should be recognized instead of being subordinate to social preferences.

Proposition 3. *Under assumption 1, Weak Internal Separability implies Weak Internal Dominance, while Weak Internal Dominance does not imply Weak Internal Separability.*

Proof. See Appendix A.

Proposition 3 argues that Weak Internal Separability is not weak as Weak Internal Dominance. It states that Weak Internal Separability implies Weak Internal Dominance under assumption 1or the variety of individual preferences. Also notice that Weak Internal Dominance fails to imply Weak

Internal Separability even under assumption 1, which indicates that Weak Internal Dominance is a fairly weak axiom.

Finally, we show that Weak Internal Separability is compatible with Permutation Pareto Principle by providing an example of a social ordering that satisfies both axioms. For any continuous preference ordering R that $bP_i a$ for all $i \in N$ implies bPa for any $a,b \in X$,[8] we define a social ordering $\boldsymbol{R}_{lpo}(R)$ that satisfies the following: for any $x,y \in X^N$, $y\boldsymbol{R}_{lpo}(R)x$ if either there exists $j \in N$ such that $y_i^R I x_i^R$ for all $i < j$ and $y_j^R P x_j^R$, or $y_i^R I x_i^R$ for all $i \in N$ and $y \geq_{par} x$.

Let any continuous preference ordering R that $bP_i a$ for all $i \in N$ implies bPa for any $a,b \in X$. Note that $bR_i a$ for all $i \in N$ also implies bRa for any $a,b \in X$ because R is continuous and from the fact that each R_i for any $i \in N$ is strictly monotonic. Jang (2017) showed that $\boldsymbol{R}_{lpo}(R)$ satisfies Permutation Pareto Principle. We prove that $\boldsymbol{R}_{lpo}(R)$ satisfies Consensus, Weak Continuity, and Weak Internal Separability in the following proposition.

Proposition 4. $\boldsymbol{R}_{lpo}(R)$ *satisfies Consensus, Weak Continuity, and Weak Internal Separability.*

Proof. See Appendix A.

Ⅳ. Concluding Remarks

This paper is in line with the study of analyzing the situation in which social equity is satisfied through the axioms. The ideas that axioms contain

8) We can interpret that R 'agrees with' every individual in that if all the agents strictly prefer b to a, so does R.

are ultimately linked to the question of the role of the government and society. Thus, showing the difficulty for important axioms, such as separability and standard Pareto principles, to be compatible with each other is meaningful in public finance, especially in terms of establishing a policy philosophy by the government or society. In this paper, we show that Internal Separability and Permutation Pareto Principle, a separability principle and a restricted form of the standard Pareto principle respectively, are incompatible. For further discussion, we introduce a demanding property, Indifference Unanimity, that restricts the indifference relation of social orderings. Then we provide an impossibility result that, there is no social ordering under the heterogeneity assumption of agents' preferences that satisfies both Permutation Pareto Principle and Internal Separability, along with another restricted form of the standard Pareto principle and continuity, namely, Consensus and Weak Continuity.

We also relieve the incompatibility results by weakening Internal Separability. We introduce Weak Internal Separability and show that it is implied by Internal Separability as well as it is compatible with Permutation Pareto Principle. We also provide the significance of Weak Internal Separability, noting that Jang (2017) already introduced Weak Internal Dominance which is weaker than Internal Separability and is compatible with Permutation Pareto Principle. To be specific, unlike Weak Internal Dominance, Weak Internal Separability shows basic respect for individual preferences and is not too weak an axiom. We finally provide an example of a social ordering that satisfies Weak Internal Separability and Permutation Pareto Principle.

An open question regarding Weak Internal Separability is to study the class of all social orderings satisfying Weak Internal Separability along with restricted forms of the standard weak Pareto principle and continuity.

Studying the logical relations among Weak Internal Separability and other axioms can also be another future research topic.

References

Arrow, K. (1950), "A Difficulty in the Concept of Social Welfare," *Journal of Political Economy*, 58 (4), 328-346.

Asheim, G., T. Mitra, and B. Tungodden (2007), "A New Equity Condition for Infinite Utility Streams and the Possibility of being Paretian," In: Roemer J., Suzumura K. (eds) Intergenerational Equity and Sustainability. International Economic Association Series. Palgrave Macmillan, London.

Atkinson, A. B. (1970), "On the Measurement of Inequality," *Journal of Economic Theory*, 2, 244-263.

Banerjee, K. (2006), "On the Equity-Efficiency Trade Off in Aggregating Infinite Utility Streams," *Economics Letters*, 93 (1), 63-67.

Basu, K. and T. Mitra (2003), "Aggregating Infinite Utility Streams with Intergenerational Equity: The Impossibility of being Paretian," *Econometrica*, 71 (5), 1557-1563.

Chun, Y. (1999), "Equivalence of Axioms for Bankruptcy Problems," *International Journal of Game Theor*y, 28, 511-520.

________ (2000), "Agreement, Separability, and Other Axioms for Quasi-Linear Social Choice Problems," *Social Choice and Welfare*, 17, 507-521.

Chun, Y., I. Jang, and B.-G. Ju (2014), "Priority, Solidarity and Egalitarianism," *Social Choice and Welfare*, 43, 577-589.

Dalton, H. (1920), "The Measurement of the Inequality of Incomes," *Economic Journal*, 30 (119), 348-361.

d'Aspremont, C. and L. Gevers (1977), "Equity and Informational Basis of Collective Choice," *Review of Economic Studies*, 44, 199-209.

Diamond, P. (1965), "The Evaluation of Infinite Utility Streams," *Econometrica*, 33 (1), 170-177.

Dietrich, F. and C. List (2007), "Arrow's Theorem in Judgment Aggregation," *Social Choice and Welfare*, 29, 19-33.

Dubey, R. and T. Mitra (2014), "Combining Monotonicity and Strong Equity: Construction and Representation of Orders on Infinite Utility Streams," *Social Choice and Welfare*, 43, 591-602.

Fleurbaey, M. (2005), "The Pazner-Schmeidler Social Ordering: A Defense," *Review of Economic Design*, 9, 145-166.

____________ (2007), "Two Criteria for Social Decisions," *Journal of Economic Theory*, 134 (1), 421-447.

Fleurbaey, M. and F. Maniquet (2008), "Fair Social Orderings," *Economic Theory*, 34, 25-45.

Fleurbaey, M. and A. Trannoy (2003), "The Impossibility of a Paretian Egalitarian," *Social choice and Welfare*, 21, 243-263.

Hammond, P. (1976), "Equity, Arrow's Conditions and Rawls' Difference Principle," *Econometrica*, 44 (4), 793-804.

Hardy, G., J. Littlewood, and G. Polya (1952), *Inequalities*, Cambridge University Press: London.

Hart, S. and A. Mas-Colell (1989), "Potential, Value and Consistency," *Econometrica*, 57 (3), 589-614.

Jang, I. (2017), "The Pareto Principle and Resource Egalitarianism," *Mathematical Social Sciences*, 85, 23-29.

Ju, B.-G., E. Miyagawa, and T. Sakai (2007), "Non-Manipulable Division Rules in Claim Problems and Generalizations," *Journal of Economic Theory*, 132 (1), 1-26.

Kolm, S. (1977), "Multidimensional Egalitarianisms," *Quarterly Journal of Economics*, 91 (1), 1-13.

Koshevoy, G. (1995), "Multivariate Lorenz Majorizations," *Social Choice and Welfare*, 12 (1), 93-102.

____________ (1997), "The Lorenz Zonotope and Multivariate Majorizations," *Social Choice and Welfare*, 15, 1-14.

Koshevoy, G. and K. Mosler (1996), "The Lorenz Zonoid of a Multivariate Distributions," *Journal of the American Statistical Association*, 91, 873-882.

Leigh, J. and J. Fries (1992), "Health Habits, Health Care Use and Costs in a Sample of Retirees," *Inquiry*, 29 (1), 44-54.

Leitgeb, H. and K. Segerberg (2007), "Dynamic Doxastic Logic: Why, How, and Where to?" *Synthese*, 155, 167-190.

Marshall, A., B. Arnold, and I. Olkin (1979), "Inequalities: Theory of Majorization and its Applications," Academic Press, New York.

Moreno-Ternero, J. and J. Roemer (2006), "Impartiality, Priority, and Solidarity in the Theory of Justice," *Econometrica*, 74 (5), 1419-1427.

Moulin, H. (1987), "Equal or Proportional Division of a Surplus, and Other Methods," *International Journal of Game Theory*, 16, 161-186.

__________ (2013), "Cost Sharing in Networks: Some Open Questions," *International Game Theory Review*, 15 (2), 134-144.

Moyes, P. (2012), "Comparisons of Heterogeneous Distributions and Dominance Criteria," *Journal of Economic Theory*, 147 (4), 1351-1383.

Muller, C. and A. Trannoy, "Multidimensional Inequality Comparisons: A Compensation Perspective," *Journal of Economic Theory*, 147 (4), 1427-1449.

Okasha, S. (2011), "Theory Choice and Social Choice: Kuhn Versus Arrow," *Mind*, 120 (477), 83-115.

Pigou, A. C. (1912), *Wealth and Welfare*, London: Macmillan Company.

Sen, A. (1973), *On Economic Inequality*, Oxford University Press, Oxford.

Shapley, L. (1953), "A Value for n-person Games," *Annals of Mathematical Studies*, 28, 307-317.

Sprumont, Y. (2012), "Resource Egalitarianism with a Dash of Efficiency," *Journal of Economic Theory*, 147 (4), 1602-1613.

Stegenga, J. (2013), "An Impossibility Theorem for Amalgamating Evidence," *Synthese*, 190, 2391-2411.

Svensson, L.-G. (1980), "Equity Among Generations," *Econometrica*, 48 (5), 1251-1256.

Viscusi, W and J. Hersch (2008), "The Mortality Cost to Smokers," *Journal of Health Economics*, 27 (4), 943-958.

Appendix A: Proofs

Proof of Proposition 1.

Let any social ordering $\boldsymbol{R}$ that satisfies Consensus, Internal Separability, and Permutation Pareto Principle. Let any $a,b \in X$ such that $(a,\ldots,a)\boldsymbol{I}(b,\ldots,b)$. We need to show that $aI_i b$ for all $i \in N$. Suppose conversely that $aP_j b$ for some $j \in N$. By Consensus, $aP_i b$ for all $i \in N$ implies $(a,\ldots,a)\boldsymbol{P}(b,\ldots,b)$. Because $(a,\ldots,a)\boldsymbol{I}(b,\ldots,b)$, there exists $k \in N \setminus \{j\}$ such that $bR_k a$.

Define $x^j = (a,\ldots,a,b,a,\ldots,a) \in X^N$ so that $x_i^j = a$ for all $i \in N \setminus \{j\}$ and $x_j^j = b$, and $x^k = (a,\ldots,a,b,a,\ldots,a) \in X^N$ so that $x_i^k = a$ for all $i \in N \setminus \{k\}$ and $x_k^k = b$. Because $(a,\ldots,a)\boldsymbol{I}(a,\ldots,a)$ and $(a,\ldots,a)\boldsymbol{I}(b,\ldots,b)$, by applying Internal Separability $n-1$ times, we get $(a,\ldots,a)\boldsymbol{I}x^j$ and $(a,\ldots,a)\boldsymbol{I}x^k$. From $(a,\ldots,a)\boldsymbol{I}x^j$ and $(a,\ldots,a)\boldsymbol{I}x^k$, we have $x^j\boldsymbol{I}x^k$ by the rationality of social ordering. Also note that, from $aP_j b$ and $aR_k a$, Permutation Pareto Principle implies $x^k\boldsymbol{P}x^j$, which is a contradiction.

Proof of Proposition 2.

Suppose that there is a difference among the agents' preferences and suppose conversely that there exists a social ordering $\boldsymbol{R}$ that satisfies Consensus, Weak Continuity, Internal Separability, and Permutation Pareto Principle. Then, by proposition 1, $\boldsymbol{R}$ also satisfies Indifference Unanimity. Without loss of generality, assume that R_1 and R_2 are not identical: there exist $a,b \in X$ such that $aP_1 b$ and $bR_2 a$. Also assume that all the other agents' preferences are identical to agent 1's preference, namely, $R_i = R_1$ for all $i \in N \setminus \{2\}$. Note that, by the strict monotonicity of R_1 and R_2, $a > 0$[9]

because $aP_1 b$, and $b > 0$ because $bR_2 a$. Let a continuous and monotonic binary relation R that satisfies bRa, and satisfies that for any two bundles $a', b' \in X$, $b' P_i a'$ for all $i \in N$ implies $b' P a'$. Because each R_i for $i \in N$ is continuous and R is also continuous, $b' R_i a'$ for all $i \in N$ also implies $b' R a'$. Let any $\epsilon_1 \in X$ such that $\epsilon_1 > 0$, $a - \epsilon_1 \in X$, and $bP_1(a - \epsilon_1)$, and any $\epsilon_2 \in X$ such that $\epsilon_2 > 0$, $a + \epsilon_2 \in X$, and $(a + \epsilon_2)P_2 b$.[10] Because $\epsilon_1 > 0$ and $\epsilon_2 > 0$, $bP_2(a - \epsilon_1)$ and $(a + \epsilon_2)P_1 b$. Because $R_i = R_1$ for all $i \in N \setminus \{2\}$, $bP_i(a - \epsilon_1)$ and $(a + \epsilon_2)P_i b$ for all $i \in N$. Then, by Consensus, $(b, \ldots, b)\boldsymbol{P}(a - \epsilon_1, \ldots, a - \epsilon_1)$ and $(a + \epsilon_2, \ldots, a + \epsilon_2)\boldsymbol{P}(b, \ldots, b)$.

Note that $\boldsymbol{R}$ is strictly monotonic because all the agents' preferences are strictly monotonic, and by Consensus. By the strict monotonicity of $\boldsymbol{R}$ and Weak Continuity, there exists $\epsilon \in X$ such that either $0 \leq \epsilon < \epsilon_2$ and $(a + \epsilon, \ldots, a + \epsilon)\boldsymbol{I}(b, \ldots, b)$, or $0 < \epsilon < \epsilon_1$ and $(a - \epsilon, \ldots, a - \epsilon)\boldsymbol{I}(b, \ldots, b)$. We finish the proof by showing that neither of the cases can hold. First, suppose that there exists $0 \leq \epsilon < \epsilon_2$ and $(a + \epsilon, \ldots, a + \epsilon)\boldsymbol{I}(b, \ldots, b)$. Then $(a + \epsilon)I_i b$ for all $i \in N$ holds by Indifference Unanimity. However, by $aP_1 b$ and the monotonicity of R_1, $(a + \epsilon)P_1 b$ holds, which is a contradiction. Now suppose that there exists $0 < \epsilon < \epsilon_1$ and $(a - \epsilon, \ldots, a - \epsilon)\boldsymbol{I}(b, \ldots, b)$. Then, by Indifference Unanimity, $(a - \epsilon)I_i b$ for all $i \in N$. However, by $bR_2 a$ and by the monotonicity of R_2, $bP_2(a - \epsilon)$ holds, a contradiction.

Proof of Proposition 3.

We first provide a social ordering that satisfies Weak Internal Dominance

---

9) To be accurate, $a > (0, \ldots, 0)$.

10) The existence of such ϵ_1 and ϵ_2 is easily guaranteed from the monotonicity of P_1 and P_2. For example, $bP_1(a - \epsilon_1)$ holds when $\epsilon_1 = a$ because $b > 0$, and $(a + \epsilon_2)P_2 b$ holds when $\epsilon_2 = b$ because $a + b > b$.

but fails to satisfy Weak Internal Separability. Suppose that aP_1a' and $(a,\ldots,a)\boldsymbol{I}(a',\ldots,a')$ for some $a,a'\in X$, and for any $b,b'\in X$, bR_2b' if and only if bR_1b'. Let $X_1\subseteq X$ be a strict subset of X, where both X_1 and $X\setminus X_1$ are unbounded. Define a social ordering $\boldsymbol{R}'$ that for all $x,x'\in X^N$, *i)* if either $x_1\in X_1$ or $x_1'\in X_1$ then $x\boldsymbol{R}'x'$ if and only if $x_1R_1x_1'$, *ii)* otherwise $x\boldsymbol{R}'x'$ if and only if $x_2R_2x_2'$.

To show that $\boldsymbol{R}'$ satisfies Weak Internal Dominance, let any $a'\in X$ and $x,x'\in X^N$ such that $x\boldsymbol{R}'x'$. First consider that either $x_1\in X_1$ or $x_1'\in X_1$. Then $x_1R_1x_1'$ holds by the definition of $\boldsymbol{R}'$. If $(x_1,\ldots,x_1)\boldsymbol{I}'(x_1',\ldots,x_1')$, then by letting $a\in X_1$ such that aR_1a',[11] we get $(a;x_{-1})\boldsymbol{R}'(a';x'_{-1})$. If $(x_i,\ldots,x_i)\boldsymbol{I}'(x_i',\ldots,x_i')$ for some $i\in N\setminus\{1\}$, then, because of $x_1R_1x_1'$, $(a;x_{-1})\boldsymbol{I}'(a';x'_{-1})$ holds for any $a\in X$. Now consider that $x_1\notin X_1$ and $x_1'\notin X_1$. Then $x_2R_2x_2'$ holds by the definition of $\boldsymbol{R}'$. If $(x_1,\ldots,x_1)\boldsymbol{I}'(x_1',\ldots,x_1')$, then $(a;x_{-1})\boldsymbol{R}'(a';x'_{-1})$ holds for any $a\in X_1$ such that aR_1a'. If $(x_2,\ldots,x_2)\boldsymbol{I}'(x_2',\ldots,x_2')$ then $(a;x_{-2})\boldsymbol{R}'(a';x'_{-2})$ holds for any $a\notin X$ such that aR_2a'. Finally, if $(x_i,\ldots,x_i)\boldsymbol{I}'(x_i',\ldots,x_i')$ for some $i\in N\setminus\{1,2\}$, then, because of $x_2R_2x_2'$, $(a;x_{-i})\boldsymbol{R}'(a';x'_{-i})$ holds for any $a\in X$. That is, whenever $(x_i,\ldots,x_i)\boldsymbol{I}'(x_i',\ldots,x_i')$ for any $i\in N$, we can find $a\in X$ such that $(a;x_{-i})\boldsymbol{R}'(a';x'_{-i})$, which indicates that $\boldsymbol{R}'$ satisfies Weak Internal Dominance.

To show that $\boldsymbol{R}'$ violates Weak Internal Separability, consider any $x,x'\in X^N$ and $a,a'\in X$ such that $(x_1,\ldots,x_1)\boldsymbol{I}'(x_i',\ldots,x_i')$ and $(a,\ldots,a)\boldsymbol{I}'(a',\ldots,a')$. Also assume that $x_1,x_1'\in X\setminus\{X_1\}$, $a,a'\in X_1$, and $x_2P_2x_2'$. From $x_1,x_1'\in X\setminus\{X_1\}$ and $x_2P_2x_2'$, $x\boldsymbol{P}'x'$ is satisfied. Also from $(a,\ldots,a)\boldsymbol{I}'(a',\ldots,a')$ and $a,a'\in X_1$, aI_1a' is satisfied. However, $a,a'\in X_1$ and aI_1a' implies $(a;x_{-i})\boldsymbol{I}'(a';x'_{-i})$, which violates Weak Internal Separability.

---

11) There exists such $a\in X_1$ because X_1 is unbounded and R_1 is monotonic.

We finish the proof by showing that Weak Internal Separability implies Weak Internal Dominance under assumption 1. Consider any social ordering $\boldsymbol{R}$ that satisfies Weak Internal Separability. Also consider any $i \in N$, $a' \in X$, and $x, x' \in X^N$ such that $(x_i, \ldots, x_i)\boldsymbol{I}(x_i', \ldots, x_i')$ and $x\boldsymbol{R}x'$. By Weak Internal Separability, we are done if we show that there exists $a \in X$ such that $(a, \ldots, a)\boldsymbol{I}(a', \ldots, a')$, and $aR_i a'$ if and only if $x_i R_i x_i'$. First assume that $x_i I_i x_i'$. Then, by letting $a = a'$, we get $(a, \ldots, a)\boldsymbol{I}(a', \ldots, a')$ and $aI_i a'$. Finally, and without loss of generality, assume that $x_i P_i x_i'$. Then, by assumption 1, there exists $a \in X$ such that $aP_i a'$ and $(a, \ldots, a)\boldsymbol{I}(a', \ldots, a')$.

Proof of Proposition 4.

Let any continuous preference ordering R that $bP_i a$ for all $i \in N$ implies bPa for any $a, b \in X$. To show that $\boldsymbol{R}_{lpo}(R)$ satisfies Consensus, suppose that $y_i P_j x_i$ for all $i, j \in N$. Then $y_1^R P_j x_1^R$ for all $j \in N$, thus, $y_1^R P x_1^R$, which in turn implies $y\boldsymbol{R}_{lpo}(R)x$. $\boldsymbol{R}_{lpo}(R)$ satisfies Weak Continuity from the fact that R is continuous.

For the rest of the proof, we show that $\boldsymbol{R}_{lpo}(R)$ satisfies Weak Internal Separability. Let any $k \in N$, $x, x' \in X^N$, and $a, a' \in X$ such that $(x_k, \ldots, x_k)\boldsymbol{I}_{lpo}(R)(x_k', \ldots, x_k')$, $(a, \ldots, a)\boldsymbol{I}_{lpo}(R)(a', \ldots, a')$, and either $[x_k P_k x_k'$ and $aP_k a']$, $[x_k I_k x_k'$ and $aI_k a']$, or $[x_k' P_k x_k$ and $a' P_k a]$. By the definition of $\boldsymbol{R}_{lpo}(R)$, $(x_k, \ldots, x_k)\boldsymbol{I}_{lpo}(R)(x_k', \ldots, x_k')$ and $(a, \ldots, a)\boldsymbol{I}_{lpo}(R)(a', \ldots, a')$ imply $x_k I x_k'$ and aIa'. Let $y = (a; x_{-k})$ and $y' = (a'; x'_{-k})$. We need to show that $x\boldsymbol{R}_{lpo}(R)x'$ if and only if $y\boldsymbol{R}_{lpo}(R)y'$. It is sufficient to show that $x\boldsymbol{P}_{lpo}(R)x'$ implies $y\boldsymbol{P}_{lpo}(R)y'$, and $x\boldsymbol{I}_{lpo}(R)x'$ implies $y\boldsymbol{I}_{lpo}(R)y'$.

First, suppose that $x\boldsymbol{P}_{lpo}(R)x'$. Then either there exists $j \in N$ such that $x_i^R I x_i'^R$ for all $i < j$ and $x_j^R P x_j'^R$, or $x_i^R I x_i'^R$ for all $i \in N$ and $x >_{par} x'$.

If $x_i^R I x_i'^R$ for all $i < j$ and $x_j^R P x_j'^R$ for some $j \in N$, then, because we have $x_k I x_k'$ and aIa', $y_h^R P y_h'^R$ for all $i < h$ and $y_h^R P y_h'^R$ for some $h \in N$, thus, $y\boldsymbol{P}_{lpo}(R)y'$. Now suppose that $x_i^R I x_i'^R$ for all $i \in N$ and $x >_{par} x'$. Then, because we have $x_k I x_k'$ and aIa', $y_i^R I y_i'^R$ for all $i \in N$. Moreover, either $[x_k P_k x_k'$ and $aP_k a']$, $[x_k I_k x_k'$ and $aI_k a']$, or $[x_k' P_k x_k$ and $a' P_k a]$, along with $x >_{par} x'$, implies $y >_{par} y'$. Therefore $y\boldsymbol{P}_{lpo}(R)y'$.

Finally, suppose that $x\boldsymbol{I}_{lpo}(R)x'$, which indicates that $x_i^R I x_i'^R$ and $x_i I_i x_i'$ for all $i \in N$. Notice that $x_i I_i x_i'$ for all $i \in N$ implies $x_k I_k x_k'$. Then either $[x_k P_k x_k'$ and $aP_k a']$, $[x_k I_k x_k'$ and $aI_k a']$, or $[x_k' P_k x_k$ and $a' P_k a]$ indicates that $x_k I_k x_k'$ and $aI_k a'$ must hold. Also, $x_i^R I x_i'^R$ and $x_i I_i x_i'$ for all $i \in N$ along with $x_k I_k x_k'$ and $aI_k a'$ imply $y_i^R I y_i'^R$ and $y_i I_i y_i'$ for all $i \in N$. Therefore $y\boldsymbol{I}_{lpo}(R)y'$.

파레토 원칙과 분리성 (separability) 원칙 간의 양립불가능성에 대하여

장 인 기*

논문초록

이 논문은 평등주의 연구에서 공공 재정 분배의 공정성 원칙들 간의 논리적 관계에 대하여 연구한다. 공정성의 한 줄기인 분리성 (Separability) 원칙은 표준 파레토 원칙과 일반적으로 양립이 불가능한 것으로 알려져 있는데, 이 연구는 공공 재정의 맥락에서 이러한 두 가지 원칙 사이의 논리적 관계를 조사한다. 구체적으로, 분리성 원칙 중 하나로 Internal Separability를 채택하고 표준 파레토 원칙 제한된 형태로 Permutation Pareto 원칙을 채택하여, 두 공리가 양립 불가능함을 보인다. 이 연구는 또한, 모든 사회 구성원들의 선호가 동일하지 않는 한, 제한된 형태의 연속성과 제한된 약한 표준 파레토 원칙, 그리고 위의 두 공리를 함께 만족하는 사회적 선호는 존재하지 않는다는 불가능성 정리도 도출한다. 마지막으로 Weak Internal Separability를 소개하고, 이 공리가 Permutation Pareto 원칙과 호환성이 있음과 동시에 Internal Separability보다 약한 공리임을 보임으로써 양립 불가능성을 해소한다.

핵심 주제어: 표준 파레토 원칙, 분리성 원칙, 평등주의, Internal Separability, Permutation Pareto 원칙

JEL 주제분류: D63, D71

* 가톨릭대학교 경제학과 조교수, e-mail: inkeejang@catholic.ac.kr