Promotion and Career Concerns in Public Sectors *

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Abstract

We present a simple model in which agents in public sectors determine the effort level with the prospect of promotion. Even without any increase of payment or fringe benefits, promotion can provide an incentive for a hard work because it can be a signal of their ability and a better job will be offered when they start new careers in private sectors. The outside firms who cannot observe the agents’ performance in public sectors use the promotion status to predict the agents’ ability. We point out that some results of the standard career concerns model do not hold here. Because promotion is a binary decision, an extra effort becomes effective only when the promotion would not have been made without it. Hence, dispersion of the agent’s ability or noise has additional negative effect while only signal-to-noisy ratio is important in the classical model.

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Keywords Promotion, Career Concerns, Moral Hazard

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1 Introduction

Understanding incentives is one of the most important subjects in economics. In many organizations, incentives are provided through monetary compensation. That is, better performance implies more money. A vast literature exists which has emphasized how firms design explicit contracts to induce employee to work in their interest. However, in some cases such as public agencies, financial incentives play a much more limited role. As it is stated in Dewatripont and Tirole(1999b), unlike a private company, a government agency is supposed to pursue social welfare which is hard to measure and therefore to reward directly. In other words, the outputs produced in public work are not verifiable cannot be used as the basis of a contract which is ultimately enforced by a third party. However, it is often observed that civil servants are driven to hard-working as much as private workers and even more. Then what drives them?

In his seminal paper, Holmstrom(1982) constructed a career concerns model of which the simplest version has two periods, today and tomorrow. Today’s performance, $x$, is the sum of the agent’s talent, current effort and noise. Performance is observable by everyone but not describable *ex ante* in a formal compensation contract. The agent is thus paid a fixed wage today. He exerts effort not just to maximize current pay but also to affect the perceptions of other which will determine her expected wage tomorrow. In sum, a good performance today can reveal, at least partially, the agent’s talent and therefore he will be awarded in the future by acquiring a better job.

In the standard career concerns model, it is assumed that the future employer is the same person as the current principle or they share the same information. However, it is quite a strong assumption in reality. For example, future careers of public workers are usually picked up in private sectors. It is hard to imagine that the private sector employers have full information about the agent’s previous performance in public sectors. If the future employers have a limited access to the agent’s performance, some of the results in the standard model may not hold. It is the main goal of this paper to verify this conjecture.

Specifically, we analyze the effect of different job designs on the agent’s effort level, when the current principle needs to make a decision whether he promotes the agent or not, based on the performance. The future employer can observe the promotion status, not the today’s performance. The promotion status is useful in predicting the agents’ ability, because the current principle will let the agent take the next job as long as he is expected to have high productivity. The process of expecting ability given the performance level in the standard career concerns model should be replaced by two steps here, “promotion decision based on the performance by the current principle” and “the expected ability given the promotion status by the future employers.”

We found that the sum of variances of talent and noise is important in determining the equilibrium effort level as signal-to-ratio is in the classical career concerns model. Note that because promotion

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1 According to a report to the National Assembly, from 2003 to 2008 the death of 414 public workers was determined as having resulted from overwork in Korea.

2 The average retirement age of public servants in Korea was 49 years old in 2006. Most of them should start a new career in private sectors.
is a binary choice, exerting an extra effort is useless if the promotion would have been made without it. The more distributions of talent and noise are disperse, the less frequently the even happens that the performance without extra effort is in the boundary of promotion criterion. Hence, exerting an effort becomes less effective. This observation leads us to present several job designs for which the current model predicts the different outcomes from the classical career concerns model.

Lastly, it is worthwhile to clarify the meaning of promotion. There is no clear consensus on what it means and various implication has been pursued. In particular, it is much emphasized that the goal of promotion in an organization is to provide incentives as well as to assign people to the jobs that best suit their abilities. That is, employees work hard in the hope of winning promotion to another job which entails a different set of responsibilities and compensations.

To focus on how to mitigate moral hazard and provide incentive through career concerns, we disregard all the privilege which the promoted job may be associated with, including wage rises. Moreover, we assume that there is no issue of job specific skill and the current employer just wants to promote the worker with high (general) productivity. Then the future employer can utilize the information of promotion status to determine his expectation on the agent’s ability.

This paper is not the first work by any means in which the limited information to the outsider is emphasized. For example, Waldman(1984) analyzed a two-period model with two jobs. At the end of the first period, only the incumbent firm is supposed to have acquired perfect information about its employee’s ability. The pair of job assignment and wage offer of the incumbent firm for the next period is public information so that competitors can make their own proposal to the agent dependent on it. Upgrading reveals to competitors that the promoted worker is good enough for the next job. Consequently, the incumbent sets an inefficiently high threshold for promotion. The strategic move by the current principle could add an interesting twist to the model developed here. Hence it should be an important future research topic.

TO BE ADDED.

2 Promotion Model

2.1 Setup

The simplest framework for the career concerns model is adapted here. A principal employs an agent for a task. $\theta$ is the agent’s ability which is initially unknown and follows a normal distribution of $N(0, \sigma_\theta)$. The first period output of the task is denoted by $x_1$ and it is determined by $x_1 = \theta + e + \varepsilon$ where $e$ is the effort the agent exerts and $\varepsilon$ is a random shock. $\varepsilon$ also follows a normal distribution of $N(0, \sigma_\varepsilon)$. Throughout the paper, $\theta$ and $\varepsilon$ are assumed to be independent. The agent incurs a

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3 Valsecchi (2008) surveys theoretical models of job mobility with special attention to promotion and career profiles. For example, Fairburn and Malcomson (1994, 2001) focus on the conditions which make promotion a credible incentive to exert unobservable effort in the case of a single firm employing many agents. The perspective of future upgrading is shown to be a relied upon incentive to exert effort at the first period because the works are aware that job assignment on the basis of inferred talent is a profit-maximizing strategy thereby not subject to influence activities.
cost from making an effort and the cost function is denoted by \( C(e) \). As usual, it is assumed that \( C(0) = 0, \ C' > 0, C'' > 0, \lim_{e \to 0} C'(e) = 0 \) and \( \lim_{e \to \infty} C'(e) = \infty \). While the principal observes \( x_1, \theta \) and \( e \) are unobservable and must be inferred from \( x_1 \).

In the second period, the agent can be promoted to another job. If the principal does not promote him, then the task must be performed by an outsider. Because, in this model, incentives are provided by the prospect of promotion, the agent will make no effort in the second period. Hence, \( x_2 = \theta \) if the agent is promoted and \( x_2 = \bar{\theta} \) if an outsider is hired where \( \bar{\theta} \) is the expected ability of outsiders. As 0 is the average ability of the agent, we assume \( \bar{\theta} = 0 \) unless it is explicitly mentioned otherwise. The results with \( \bar{\theta} \neq 0 \) are not much different qualitatively as long as \( \bar{\theta} \) is close enough to 0. The principle’s objective function is \( x_1 + x_2 \).

The agent will not receive any compensation in both periods or be paid the fixed amount. However, a better wage offer will be made by outside firms if he is promoted. Unlike previous literature of career concerns, the outside firms can observe neither the first period output nor the second period one. Only the promotion status will be known to them. Assuming that the outside labor market is perfectly competitive, the offered wage must be equal to the expected ability given the promotion status. The timing of the game is summarized as follows.

1. The decision on \( e \) is made.
2. \( \theta \) and \( \varepsilon \) are realized.
3. \( x_1 \) is obtained.
4. The promotion decision is made by the principle after observing \( x_1 \).
5. \( x_2 \) is obtained.
6. The outside firms make a wage offer to the promoted agent(s).

In the standard career concerns model, the outside firm can observe the first period output and there is no second period such that promotion is not an issue. For the future reference, we are going to call it the model with direct observation.

### 2.2 Ability Distribution and Signal-to-Noise Ratio

Let \( \mu \) be the additional benefit to the agent from the better job offer in case of promotion. Because the outside labor market is perfectly competitive, the offered wage must be equal to the expected ability given the promotion status. Then \( \mu = E_o(\theta | I = 1) - E_o(\theta | I = 0) \) where \( I \) is the indicator function showing promotion and the subscript \( o \) means that it is the expectation of the outside firms. Let the probability of promotion and the equilibrium effort level be denoted by \( P(e) \) and \( e^* \),
respectively. Then the agent’s payoff becomes

\[ -C(e) + P(e)E_o (\theta_A \mid I_A = 1) + (1 - P(e))E_o (\theta_A \mid I_A = 0) \]

\[ = -C(e) + P(e)\mu + E_o (\theta_A \mid I_A = 0). \]

Along the equilibrium, the principle and the outside firms perfectly expect \( e^* \) though they cannot observe the actual effort level. The promotion decision given \( x_1 \) should be

\[
\begin{cases}
E[\theta \mid x_1 - e^*] \geq \bar{\theta} = 0: \text{the agent is promoted} \\
E[\theta \mid x_1 - e^*] < \bar{\theta} = 0: \text{the agent is not promoted}
\end{cases}
\]

\[ \Rightarrow E(\theta) + \frac{Cov[\theta, x_1 - e^*]}{Var[x_1 - e^*]} (x_1 - e^*) \geq 0: \text{the agent is promoted} \]

\[ \Rightarrow \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} (\theta + \varepsilon + e - e^*) \geq 0: \text{the agent is promoted} \]

The outside firm’s expectation about the agent’s ability along the equilibrium is

\[ E_o[\theta \mid I = 1] = E_o \left[ \theta \mid \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} (\theta + \varepsilon) \geq 0 \right] = \sqrt{\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}} \frac{\phi(0)}{1 - \Phi(0)}, \]

\[ E_o[\theta \mid I = 0] = E_o \left[ \theta \mid \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} (\theta + \varepsilon) \leq 0 \right] = -\sqrt{\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}} \frac{\phi(0)}{1 - \Phi(0)} \]

\[ \Rightarrow \mu = \sqrt{\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}} \frac{2\phi(0)}{1 - \Phi(0)} \]

(1)

where \( \phi \) and \( \Phi \) are the density function and the cumulative density function of the standard normal distribution, respectively. Then \( P(e) \) given \( e^* \) is driven as follows.

\[ \text{The derivation in detail is given in the appendix.} \]
\[ P(e) = \Pr_{\theta+\varepsilon}[E[\theta \mid x_1 - e^*] \geq 0] \]
\[ = \Pr_{\theta+\varepsilon}\left[\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_z^2}(x_1 - e^*) \geq 0\right] \]
\[ = \Pr_{\theta+\varepsilon}\left[\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_z^2}(\theta + \varepsilon) + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_z^2}(e - e^*) \geq 0\right] \]
\[ = \Pr_{\theta+\varepsilon}[\theta + \varepsilon \geq e^* - e] \]
\[ = \Pr_{\theta+\varepsilon}\left[\frac{\theta + \varepsilon}{\sqrt{\sigma_\theta^2 + \sigma_z^2}} \geq \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_z^2}}(e^* - e)\right] \]
\[ = \int_0^\infty \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_z^2}}(e^* - e) \ d\Phi(z) \]
\[ \Rightarrow P'(e^*) = \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_z^2}} \phi(0) \]

Note that \( P'(e^*) \) is decreasing in \( \sigma_\theta^2 \) and \( \sigma_z^2 \). It means that the marginal increase of promotion probability from an extra effort is lower when the agent’s ability or noise is more dispersed. A larger effort level than the equilibrium one will lead to a larger output at any given \( \theta + \varepsilon \). However, it does not always affect the promotion decision, because the promotion is binary; success or fail. For example, if \( \theta + \varepsilon > 0 \), then making an extra effort is useless because the promotion would be made without it anyway. In other words, exerting an extra effort is effective only when \( \theta + \varepsilon = 0 \) by chance. When the dispersion of \( \varepsilon \) or \( \theta \) is larger, then the probability of \( \theta + \varepsilon = 0 \) is lower and so is \( P'(e^*) \), which is the key observation in this paper.

The objective function of the agent, \(-C(e) + \mu P(e) + E_o[\theta \mid I = 0]\), should be maximized at \( e = e^* \). The first order condition is

\[ C'(e^*) = \mu P'(e^*) \]
\[ = 2 \frac{\sigma_\theta}{\sigma_\theta^2 + \sigma_z^2} \frac{\phi(0)^2}{1 - \Phi(0)} \]

Note that \( \mu \) and \( E_o[\theta \mid I = 0] \) are not a function of \( e \) but \( e^* \). Even when the agent takes a different effort level from \( e^* \), the outside firms calculate their expectation based on \( e^* \). Proposition 1 compares the equilibrium effort level here with that of the standard career concerns model in which the outside firm can directly observe the first period output.5

**Proposition 1** The equilibrium effort level in this model is lower (higher) than that of the direct observation model if \( \frac{2\phi(0)^2}{1 - \Phi(0)} < (> \sigma_\theta \).

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5We also need to assume that the second period does not exist or its performance is unobservable in the standard career concerns model.
Proof: If the firm can observe the first period output $x_1$ and there is no second period, the outside firm will offer $E[\theta \mid x_1 - e^*] = \frac{\sigma^2}{\sigma^2 + \sigma^2_\varepsilon} (x_1 - e^*)$. Then the condition for the optimal effort level is
\[
\frac{dE}{de} \left[ \frac{\sigma^2}{\sigma^2 + \sigma^2_\varepsilon} (\theta + \varepsilon + e - e^*) \right]_{e=e^*} = C'(e^*)
\Rightarrow \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\varepsilon} = C'(e^*)
\]
Note that $C'$ is an increasing function in $e$. The result in the statement is obtained by comparing (2) and (3). ■

Proposition 2 below observes that the equilibrium effort level can be increasing or decreasing, depending on the relative dispersion between ability and noise.

**Proposition 2**

\[
\frac{de^*}{d\sigma_\theta} < 0 \text{ if } \sigma^2_\varepsilon < \sigma^2_\theta \\
\frac{de^*}{d\sigma_\theta} > 0 \text{ if } \sigma^2_\varepsilon > \sigma^2_\theta
\]

*Proof:* It is clear from (2). ■

In particular, the result of $\frac{de^*}{d\sigma_\theta} < 0$ is interesting. $\frac{de^*}{d\sigma_\theta} < 0$ never arises in the direct observation model. $\frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\varepsilon}$ in (3) is called signal-to-noise ratio. If the first period output $x_1$ is different from the expected effort level $e^*$, the outside firms attribute $x_1 - e^*$ to both the agent’s ability and pure noise. $\frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\varepsilon}$ is the portion of $x_1 - e^*$ which goes to ability. With an increase of the variance of the agent’s ability, the signal-to-noise ratio rises and it helps him to receive the better wage offer given any $x_1$. Hence, the incentive to exert extra effort becomes larger in the standard model.

However, in this promotion model, the benefit of making an extra effort to the agent is $\mu P'(e)$ which consists of $\mu$ and $P'(e)$. An increase of the variance of ability affects both and the direction is the opposite. The effect on the marginal increase of the promotion probability, $P'(e)$, is always negative as it is explained before. The effect on $\mu$ is positive because of the larger signal-to-noise ratio. If $\sigma^2_\varepsilon < \sigma^2_\theta$, then the first effect dominates the second one and the equilibrium effort level drops as the agent’s ability is more disperse.

### 2.3 Working Together

One of the applications of Proposition 2 is a “team work”. Suppose there are two agents, $A$ and $B$. In a team project, the task must be performed by both agents, that is, $x_1 = \theta_A + e_A + \theta_B + e_B + \varepsilon$. The meanings of $\theta_A, \theta_B, e_A$ and $e_B$ are self-explanatory. We assume $(\theta_A, \theta_B)'$ follows a multi-variate
normal distribution of $N((0, 0)', \Sigma_b)$ where $\Sigma_b = \begin{pmatrix} \sigma_\theta^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_\theta^2 \end{pmatrix}$ and $\sigma_{ab} \geq 0$. To focus on ability distribution, we assume that the promotion decision and hiring of the outside firms must be made as a team. It means that $A$ and $B$ will be promoted (hired) together or not. However, the agents determine their effort level non-cooperatively. The promotion decision is given as follows.

$$E(\theta_A + \theta_B \mid x_1 - e^*_A - e^*_B \geq 0) : \text{The team is promoted}$$

$$\Rightarrow E(\theta_A + \theta_B \mid \theta_A + \theta_B + e_A - e^*_A + e_B - e^*_B + \varepsilon) \geq 0$$

$$\Rightarrow \frac{2\sigma_\theta^2 + 2\sigma_{ab}}{2\sigma_\theta^2 + 2\sigma_{ab} + \sigma_\varepsilon^2} (\theta_A + \theta_B + e_A - e^*_A + e_B - e^*_B + \varepsilon) \geq 0$$

The promotion probability of $A$ given $(e_A, e^*_B)$ is

$$P_A(e_A, e^*_B) = \Pr[\theta_A + \theta_B + e_A - e^*_A + \varepsilon \geq 0]$$

$$\Rightarrow \frac{\partial P_A(e_A, e^*_B)}{\partial e_A} \bigg|_{e_A = e^*_A} = \frac{\phi(0)}{\sqrt{2\sigma_\theta^2 + 2\sigma_{ab} + \sigma_\varepsilon^2}}$$

The outside firms' expectation on the team's ability given promotion is

$$E_o(\theta_A + \theta_B \mid I_A = 1) = E_o(\theta_A + \theta_B \mid x_1 - e^*_A - e^*_B \geq 0)$$

$$= E_o(\theta_A + \theta_B \mid \theta_A + \theta_B + \varepsilon \geq 0)$$

$$= \frac{2\sigma_\theta^2 + 2\sigma_{ab}}{\sqrt{2\sigma_\theta^2 + 2\sigma_{ab} + 2\sigma_{ab} + \sigma_\varepsilon^2}} \frac{\phi(0)}{1 - \Phi(0)}.$$}

Then the equilibrium condition is

$$\mu \frac{\partial P_A(e_A, e^*_B)}{\partial e_A} \bigg|_{e_A = e^*_A} = C'(e^*_A)$$

$$\Rightarrow \frac{2\sqrt{2\sigma_\theta^2 + 2\sigma_{ab}} \phi(0)^2}{(2\sigma_\theta^2 + 2\sigma_{ab} + \sigma_\varepsilon^2)(1 - \Phi(0))} = C'(e^*_A).$$

Note that $\frac{\sqrt{2\sigma_\theta^2 + 2\sigma_{ab}}}{(2\sigma_\theta^2 + 2\sigma_{ab} + \sigma_\varepsilon^2)} < \frac{\sigma_\theta}{\sigma_\theta + \sigma_\varepsilon}$ if $\sigma_\varepsilon^2 < \sigma_\theta^2$. Hence, we conclude Proposition 3 below.

**Proposition 3** The agent’s effort level is lower in working as a team than that of single agent case if $\sigma_\varepsilon^2 < \sigma_\theta^2$.
evaluated as a team let signal-to-noise ratio be favorable and the equilibrium effort will be larger than that of single agent case in the direct observation model. However, there is another negative effect in this model because it lowers the probability of the event that exerting an additional effort is effective. Under the condition of $\sigma^2_e < \sigma^2_\theta$, the negative effect dominates the positive one and the equilibrium effort level drops, contrary to one in the direct observation model.

3 Correlated Performance

3.1 Competition for Promotion

In this subsection, we consider the case of two agents, $A$ and $B$, and two missions, $X$ and $Y$. The first period outputs in $X$ and $Y$ are determined by $x_1 = \theta_A + e_A + \varepsilon^X$ and $y_1 = \theta_B + e_B + \varepsilon^Y$. $(\theta^a, \theta^b)$ follows a multi-variate normal distribution of $(\theta^a, \theta^b) \sim N((0, 0)', \Sigma_{\theta})$ where $
abla_{\theta_1} \nabla_{\theta_2} \Sigma_{\theta} = \begin{pmatrix} \sigma^2_\theta & \sigma_{ab} \\ \sigma_{ab} & \sigma^2_\theta \end{pmatrix}$. $(\varepsilon^X, \varepsilon^Y)$ also follows another multi-variate normal distribution of $(\varepsilon^X, \varepsilon^Y) \sim N((0, 0)', \Sigma_{\varepsilon})$ where $
abla_{\varepsilon_1} \nabla_{\varepsilon_2} \Sigma_{\varepsilon} = \begin{pmatrix} \sigma^2_\varepsilon & \sigma_{xy} \\ \sigma_{xy} & \sigma^2_\varepsilon \end{pmatrix}$. We continue to assume that $(\theta^a, \theta^b)$ and $(\varepsilon^X, \varepsilon^Y)$ are independent. In addition, there is only one promotion job for the agents and no outside option is available to the principal. That is, either $A$ or $B$ must be promoted in the second period. Two agents’ cost functions are identical.

The condition of $A$’s promotion is given below.

$$E(\theta_A | x_1 - e^*_A, y_1 - e^*_B) \geq E(\theta_B | x_1 - e^*_A, y_1 - e^*_B)$$

$$\Rightarrow \begin{pmatrix} (\sigma^2_\theta, \sigma_{ab})Var[x_1 - e^*_A, y_1 - e^*_B]^{-1} \\ (x_1 - e^*_A, y_1 - e^*_B) \end{pmatrix} \geq \begin{pmatrix} (\sigma^2_\theta, \sigma_{ab})Var[x_1 - e^*_A, y_1 - e^*_B]^{-1} \\ (x_1 - e^*_A, y_1 - e^*_B) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \sigma^2_\theta(\sigma^2_\theta + \sigma^2_e) \\ -\sigma_{ab}(\sigma_{xy} + \sigma_{ab}) \\ +\sigma^2_\theta(\sigma^2_\theta + \sigma^2_e) \end{pmatrix} (x_1 - e^*_A) + \begin{pmatrix} \sigma_{ab}(\sigma^2_\theta + \sigma^2_e) \\ -\sigma^2_\theta(\sigma_{xy} + \sigma_{ab}) \\ +\sigma_{ab}(\sigma^2_\theta + \sigma^2_e) \end{pmatrix} (y_1 - e^*_B) \geq \begin{pmatrix} \sigma_{ab}(\sigma^2_\theta + \sigma^2_e) \\ -\sigma^2_\theta(\sigma_{xy} + \sigma_{ab}) \\ +\sigma_{ab}(\sigma^2_\theta + \sigma^2_e) \end{pmatrix} (x_1 - e^*_A)$$

$$\Rightarrow x_1 - e^*_A \geq y_1 - e^*_B$$

The probability of each agent’s promotion and its derivative with respect to effort level are obtained as follows.
They showed that the effort level with the comparative performance scheme is lower (higher) than
and

\[ E \]

\[ \text{effort level in the direct observation model in which the agent} \]

\[ \text{agent case if} \]

\[ \text{Proposition 4} \]

\[ \text{The outside firms’ wage offer to the promoted agent is:} \]

\[ E_o (\theta_A | I_A = 1) = E_o (\theta_A | \theta_A + \varepsilon^X \geq \theta_B + \varepsilon^Y) \]

\[ = \frac{\sigma_{e_a} - \sigma_{ab}}{\sqrt{\sigma_{e_a}^2 + 2\sigma_{e_a}^2 - 2\sigma_{xy} - 2\sigma_{ab}}} \phi (0) \]

\[ \Rightarrow \mu_A = \frac{2\sigma_{e_a} - 2\sigma_{ab}}{\sqrt{2\sigma_{e_a}^2 + 2\sigma_{z}^2 - 2\sigma_{xy} - 2\sigma_{ab}}} \phi (0) \]

The equilibrium conditions become

\[ \mu_A \frac{\partial P_A (e_A, e_B)}{\partial e_A} |_{e_A = e_A^*} = \frac{\sigma_{\theta} - \frac{\sigma_{ab}}{\sigma_{\theta}}}{\sigma_{\theta}^2 + \sigma_{z}^2 - \sigma_{xy} - \sigma_{ab}} \phi (0) = C' (e_A^*) \]

\[ \mu_B \frac{\partial P_B (e_A, e_B)}{\partial e_B} |_{e_B = e_B^*} = \frac{\sigma_{\theta} - \frac{\sigma_{ab}}{\sigma_{\theta}}}{\sigma_{\theta}^2 + \sigma_{z}^2 - \sigma_{xy} - \sigma_{ab}} \phi (0) = C' (e_B^*). \]

**Proposition 4 The agent’s effort level of this subsection model is lower (higher) than that of single agent case if**

\[ (\sigma_{\theta}^2 + \sigma_{z}^2) - \frac{\sigma_{ab}}{\sigma_{\theta}} (\sigma_{\theta}^2 - \sigma_{xy}) - 2\sigma_{xy} > 0 (< 0). \]

**Proof:** Let us compare (2) and (4) or (5). It is straightforward because

\[ \frac{\sigma_{\theta} - \frac{\sigma_{ab}}{\sigma_{\theta}}}{\sigma_{\theta}^2 + \sigma_{z}^2 - \sigma_{xy} - \sigma_{ab}} - \frac{2\sigma_{\theta}}{\sigma_{\theta}^2 + \sigma_{z}^2} < 0 \leftrightarrow (\sigma_{\theta}^2 + \sigma_{z}^2) - \frac{\sigma_{ab}}{\sigma_{\theta}} (\sigma_{\theta}^2 - \sigma_{xy}) - 2\sigma_{xy} > 0 (< 0) \]

Meyer and Vickers (1997) analyzed the comparative performance and its effect on the agent’s effort level in the direct observation model in which the agent A and B will be paid \( E (\theta_A | x_1 - e_A^*, y_1 - e_B^*) \) and \( E (\theta_B | x_1 - e_A^*, y_1 - e_B^*) \), respectively.

They showed that the effort level with the comparative performance scheme is lower (higher) than
that of single agent one if
\[
\left( \frac{\sigma_0^2}{\sigma_0^2 + \sigma_z^2} \sigma_{ab} + \frac{\sigma_0^2}{\sigma_0^2 + \sigma_z^2} \sigma_{xy} \right) (\sigma_{xy} - \sigma_{ab}) < (>) 0. \tag{7}
\]

To understand the difference between (6) and (7), for example, assume \( \sigma_{xy} \neq 0 \) and \( \sigma_{ab} = 0 \). Then
\[
\left( \frac{\sigma_0^2}{\sigma_0^2 + \sigma_z^2} \sigma_{ab} + \frac{\sigma_0^2}{\sigma_0^2 + \sigma_z^2} \sigma_{xy} \right) (\sigma_{xy} - \sigma_{ab}) > 0 \text{ in (7).}
\]
The principle uses both performance, \( x \) and \( y \), in prediction \( \theta_A \). Because \( \epsilon^y \) in \( y \) is correlated with \( \epsilon^x \) in \( x \) and \( \theta_B \) is independent of \( \theta_A \), the additional information from \( y_1 \) decreases the effective variance of noise for \( A \) and so decreases the weight on it.

Then the agent’s effort level is higher than independent evaluation.

Meanwhile, \( \sigma_{ab} = 0 \) implies that the sign of \( \sigma_0^2 + \sigma_z^2 - \frac{\sigma_{ab}^2}{\sigma_{xy}} \sigma_{xy} \) in (6) depends on the sign of \( \sigma_0^2 + \sigma_z^2 - 2 \sigma_{xy} \). \( \sigma_0^2 + \sigma_z^2 - 2 \sigma_{xy} > 0 \) makes the probability of the event that exerting an additional effort is effective become lower. In addition, under \( \sigma_0^2 + \sigma_z^2 - 2 \sigma_{xy} > 0 \), the existence of \( B \) affects \( \mu \) negatively and the effort level drops in this model.

Suppose now that \( \sigma_{xy} = 0 \) and \( \sigma_{ab} \neq 0 \), the effort level will decrease in the performance observation model because the correlation of two abilities of \( A \) and \( B \) effectively decreases the weight on ability. In this promotion model, \( \sigma_{xy} = 0 \Rightarrow (\sigma_0^2 + \sigma_z^2) - \frac{\sigma_{ab}^2}{\sigma_{xy}} (\sigma_0^2 + \sigma_z^2 - 2 \sigma_{xy} > 0 \) in (6) and the effort level decreases. It happens because the negative effects dominates the positive one.

### 3.2 Comparative Performance

The previous subsection assumes there is only one promoted job and competition between two agents exist. This assumption does not derive the main conclusion though the quantitative outcome is, of course, different without it. In this subsection we continue to assume that there are two agents and two missions. However, there are a promoted job for each agent and the outside options are also available. Therefore these are all possible that both are promoted, neither is promoted or only one of two is promoted. To avoid messy algebra, we are going to assume that \( \sigma_{xy} = \sigma_{ab} = z \).\(^6\) Because two performances are still correlated, the principle should use a comparative performance criterion. The condition of promoting \( A \) is given as follows.

\[
|E(\theta_A | x_1 - e_A^*, y_1 - e_B^*)| \geq 0 \Rightarrow \langle \sigma_0^2, z \rangle Var[x_1 - e_A^*, y_1 - e_B^*]^{-1} \left( \begin{array}{c} x_1 - e_A^* \\ y_1 - e_B^* \end{array} \right) \geq 0 \\
\Rightarrow \langle \sigma_0^2(\sigma_0^2 + \sigma_z^2 - 2z^2)(x_1 - e_A^* + (z(\sigma_0^2 + \sigma_z^2 - 2\sigma_0^2 z)(y_1 - e_B^*) \geq 0 \\
\Rightarrow \langle x_1 - e_A^* + \frac{N}{M}(y_1 - e_B^*) \geq 0
\]

\(^6\)Keep in mind that our main goal is to compare the model with the performance observation one. The case of \( \sigma_{xy} = \sigma_{ab} \) is one example showing the results in two cases can be different.
where \( M = \sigma^2_{\theta} (\sigma^2_{e} + \sigma^2_{\epsilon}) - 2z^2 \) and \( N = z (\sigma^2_{e} - \sigma^2_{\theta}) \). Then,

\[
P_A(e_A, e_B) = \Pr \left[ \frac{x_1 - e_A^*}{M} + \frac{N}{M} (y_1 - e_B^*) \geq 0 \right]
\]

\[
\Rightarrow \frac{\partial P_A(e_A, e_B^*)}{\partial e_A} \bigg|_{e_A = e_A^*} = \frac{\phi(0)}{\sqrt{1 + \frac{N^2}{M^2} (\sigma^2_{\theta} + \sigma^2_{\epsilon}) + 4 \frac{N}{M} z}}
\]

The outside firms’ wage offer to promoted \( A \) is:

\[
E_o (\theta_A \mid I_A = 1) = E_o \left( \theta_A \mid (\theta_A + \epsilon^X) + \frac{N}{M} (\theta_B + \epsilon^Y) \geq 0 \right)
\]

\[
\Rightarrow \mu_A = \frac{2 (\sigma^2_{\theta} + \frac{N}{M} z)}{\sqrt{\sigma^2_{\theta}} \sqrt{1 + \frac{N^2}{M^2} (\sigma^2_{\theta} + \sigma^2_{\epsilon}) + 4 \frac{N}{M} z}} \frac{\phi(0)}{1 - \Phi (0)}
\]

The equilibrium condition becomes:

\[
\mu_A \frac{\partial P_A(e_A, e_B^*)}{\partial e_A} \bigg|_{e_A = e_A^*} = C'(e_A^*)
\]

\[
\Rightarrow \frac{2 (\sigma^2_{\theta} + \frac{N}{M} z)}{\sqrt{\sigma^2_{\theta}} \left[ (1 + \frac{N^2}{M^2}) (\sigma^2_{\theta} + \sigma^2_{\epsilon}) + 2 \frac{N}{M} z \right]} \frac{\phi(0)^2}{1 - \Phi (0)} = C'(e_A^*)
\]

\[
\Rightarrow \frac{2 (\sigma^2_{\theta} + \frac{N}{M} z \sigma_{\epsilon})}{(1 + \frac{N^2}{M^2}) (\sigma^2_{\theta} + \sigma^2_{\epsilon}) + 4 \frac{N}{M} \sigma_{ab} 1 - \Phi (0)} = C'(e_A^*)
\]

**Proposition 5** The effort level in the model of this subsection is lower (higher) than that of single agent case if \( \sigma^2_{\theta} > \sigma^2_{\epsilon} \) (\( \sigma^2_{\theta} < \sigma^2_{\epsilon} \)).

**Proof**: In the appendix

As it is explained before, the condition in the performance observation model of Meyer and Vickers (1997) is \( \left( \frac{\sigma^2_{\theta}}{\sigma^2_{\theta} + \sigma^2_{xy}} \sigma_{ab} + \frac{\sigma^2_{\epsilon}}{\sigma^2_{\theta} + \sigma^2_{xy}} \sigma_{xy} \right) (\sigma_{xy} - \sigma_{ab}) > (<) 0 \). With \( \sigma_{xy} = \sigma_{ab} \), the equilibrium effort will be exactly same as the one without comparative performance criterion. However, in this promotion model, it depends on the sign of \( \sigma^2_{\theta} - \sigma^2_{\epsilon} \).

4 Conclusion

The design of incentives, information availability and contractibility are closed related. When the principle cannot form a contract based on current output due to its non-contractibility, other ways of providing an incentive must be developed. Career concerns model captures a motivation of work, in which good performance today can be a signal of high productivity. Then he can be rewarded later through a better job offer by the future potential employers. This approach assumes that the future
employer can observe today’s performance and, therefore he takes the expectation about worker’s ability based on it.

This paper starts with a simple observation that the future employer usually has a limited access to the outcome of the current, in reality. In particular, the objectives of the public service is very abstract and evaluation for public servants mainly consists of subjective satisfaction of the superior. Moreover the government itself, sometimes, is reluctant to reveal all the relevant information about the agencies’ performance because of political concern. However, it is well known and easy to acquire that what kind position one has taken or what class he belonged to before retirement.

What should be the effect of different job designs on the agent’s effort level when promotion is determined by the current employer but the future job offer is made only based on the promotion status by the outsider? In the standard career concerns model, the key parameter is signal-to-noise ratio which is basically the variance of ability over the variance of noise. If this ratio is high, then the future employers attribute good performance a lot to ability effect and, therefore an incentive to raise the current outcome through higher effort becomes larger. We found that, instead, the sum of variance of ability and noise can be the key parameter value here.

The benefit from an extra effort to the agent can be realized through two steps. The first one is “promotion” and the other one is ‘the expected ability given promotion.” The latter is determined by signal-to-noise ratio as in the standard career concerns model. However, the first one depends on the sum of the variances of two variables, ability and noise. Because promotion is a binary choice (success/failure), exerting an extra effort becomes beneficial only when the promotion would not have been made without it. Hence, dispersion of the two variables will make negative effect because it makes the event rare that the promotion would be just made without the extra effort. We provides several situations that this additional effect leads us to the different results from the classical career concerns model. This observation helps us to understand the job design of public sectors better.

TO BE ADDED.
5 Appendix

Derivation of $E(\theta \mid \delta \geq \rho)$

Let $\delta = k_1 \theta + \gamma$ where $(\theta, \gamma)' \sim N((0, 0)', \Sigma)$ where $\Sigma = \begin{pmatrix} \sigma^2_\theta & \sigma_\theta \gamma \\ \sigma_\theta \gamma & \sigma^2_\gamma \end{pmatrix}$ and $0 < k_1$. Let $\Delta = \frac{\delta}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}} = \frac{k_1 \theta + \gamma}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}}$. Then $\Delta \sim N(0, 1)$. Let’s decompose $\theta$ into $\Delta$ and the error term, $z$, which is orthogonal to $\Delta$. You may interpret this as regressing $\Delta$ into $\theta$, that is, $\theta = \lambda \Delta + \sqrt{1 - \lambda^2}z$ where $\Delta$ and $z$ are independent and $\lambda$ is the correlation coefficient between $\theta$ and $\Delta$. Then,

$E(\theta \mid \delta \geq \rho)$

$= E \left( \frac{\rho}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}} \right)$

$= E \left( \lambda \Delta + \sqrt{1 - \lambda^2}z \mid \Delta \geq \frac{\rho}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}} \right)$

$= \lambda E \left( \frac{\rho}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}} \right)$

$= \phi \left( \frac{\rho}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}} \right)$

$\frac{\sqrt{\text{Var(}\theta)} \sqrt{\text{Var(}\Delta)}}{\text{Cov(}\theta, \Delta)} \phi \left( \frac{\rho}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}} \right)$

$= \frac{k_1 \sigma^2_\theta + \sigma_\gamma}{\sqrt{\sigma^2_\theta}} \phi \left( \frac{\rho}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}} \right)$

$= \frac{k_1 \sigma_\theta + \frac{\sigma_\gamma}{\sigma_\theta}}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}} \phi \left( \frac{\rho}{\sqrt{k_1^2 \sigma^2_\theta + \sigma^2_\gamma + 2k_1 \sigma_\theta \gamma}} \right)$

Proof of Proposition 4

13
Let us compare (2) and (6).

\[
\frac{2 \left( \sigma_\theta + \frac{N}{M} z \right)}{(1 + \frac{N^2}{M^2}) (\sigma^2_\theta + \sigma^2_z) + 4 \frac{N}{M} z} - \frac{2\sigma_\theta}{\sigma^2_\theta + \sigma^2_z} = 2 \left( \sigma^2_\theta + \frac{N}{M} z \right) (\sigma^2_\theta + \sigma^2_z) - \sigma^2_\theta \left[ (1 + \frac{N^2}{M^2}) (\sigma^2_\theta + \sigma^2_z) + 4 \frac{N}{M} z \right] \sigma_\theta \left[ (1 + \frac{N^2}{M^2}) (\sigma^2_\theta + \sigma^2_z) + 4 \frac{N}{M} z \right] (\sigma^2_\theta + \sigma^2_z) = \frac{N}{M} \sigma_\theta \left( (1 + \frac{N^2}{M^2}) (\sigma^2_\theta + \sigma^2_z) + 4 \frac{N}{M} \sigma_\theta \right) (\sigma^2_\theta + \sigma^2_z) = \frac{z(\sigma^2_\theta - \sigma^2_z)}{M} \left( \sigma^2_\theta + \sigma^2_z \right) \frac{2z(\sigma^2_\theta - z^2)}{\sigma^2_\theta} - 4\sigma^2_z = \frac{2z^2(\sigma^2_\theta - \sigma^2_z)}{M} \frac{\sigma^2_\theta \left( (1 + \frac{N^2}{M^2}) (\sigma^2_\theta + \sigma^2_z) + 4 \frac{N}{M} \sigma_\theta \right) (\sigma^2_\theta + \sigma^2_z)}{\sigma^2_\theta \left( (1 + \frac{N^2}{M^2}) (\sigma^2_\theta + \sigma^2_z) + 4 \frac{N}{M} \sigma_\theta \right) (\sigma^2_\theta + \sigma^2_z)}
\]

Let \( \Lambda = -\sigma^4_\theta (\sigma^2_\theta + \sigma^2_z) + z^2(3\sigma^2_\theta - \sigma^2_z) \). Then

\[
\Lambda = -\sigma^4_\theta (\sigma^2_\theta + \sigma^2_z) + z^2(3\sigma^2_\theta - \sigma^2_z) = \sigma^2_\theta (z^2 - \sigma^4_\theta) + \sigma^2_\theta (z^2 - \sigma^4_\theta \sigma^2_z) + z^2(\sigma^2_\theta - \sigma^2_z)
\]

If \( \sigma^2_\theta < \sigma^2_z \), then \( \Lambda < 0 \) because \( z^2 \leq \min \{ \sigma^4_\theta, \sigma^2_\theta \sigma^2_z \} \). Hence we obtain the half of the conclusion above. In addition, if \( \sigma^2_\theta > \sigma^2_z \), then

\[
\Lambda = \sigma^2_\theta (z^2 - \sigma^4_\theta) + \sigma^2_\theta (z^2 - \sigma^4_\theta \sigma^2_z) + z^2(\sigma^2_\theta - \sigma^2_z) < \sigma^2_\theta (z^2 - \sigma^4_\theta) + z^2(\sigma^2_\theta - |z|) = (\sigma^2_\theta - |z|)(-\sigma^4_\theta - \sigma^2_\theta \sigma^2_z) \leq 0
\]

which proves the other half of the conclusion. \( \blacksquare \)
Derivation of ??

\[
\begin{align*}
\text{Var}[x_1 - e^*_A, y_1 - e^*_B]^{-1} &= \\
&= \frac{1}{\left(\sigma^2_\theta + \sigma^2_\varepsilon\right)^2 - (\sigma_{ab} + \sigma_{xy})^2} \\
&\quad \left[ \begin{array}{cc}
\sigma^2_\theta + \sigma^2_\varepsilon, & -\sigma_{ab} - \sigma_{xy} \\
-\sigma_{ab} - \sigma_{xy}, & \sigma^2_\theta + \sigma^2_\varepsilon
\end{array} \right] \\
&\quad \left( x_1 - e^*_A \right) \\
&\quad \left( y_1 - e^*_B \right)
\end{align*}
\]
References


