Regulation of Natural Monopolies, Self-Selection, and Optimal Taxation: Theory and empirical evidence*

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March 2012

Abstract

It is well-known that the increasing returns-to-scale (IRS) property accounts for the presence of natural monopolies, which usually become public enterprises or are subject to regulations. This paper argues that public enterprises provide private goods not only for the IRS property, but also for relaxing the incentive problem of the tax system: they help relax the self-selection constraint of the optimal income tax problem through nonlinear pricing. The intuition is that when some private goods with IRS properties (e.g., public transportation) relative to other goods are more valuable to low-ability individuals than the high-ability counterparts in terms of the marginal rate of substitution (MRS), the high-ability individuals are discouraged to mimic low-ability ones. Our results provide theoretical underpinning for the low price of publicly provided private goods for low-income individuals, breaking the \( p = MC \) rule for efficient redistribution. The optimal nonlinear pricing allows low marginal tax rates for both types, leading to greater work incentives. We implement this idea empirically and obtain supportive evidence using individual panel data.

Key Words: public provision of private goods, self-selection and optimal taxation

JEL Codes: H21,H42

[Final draft; comments welcome!]

*Acknowledgments: This paper was initially prepared for presentation at the thirtieth anniversary meeting of the Korea Association of Public Finance. The authors are grateful for useful comments and discussions to Roger Gordon and conference participants at KAPF and AEI in Singapore. Lee: leeci@snu.ac.kr; Ko: jxko1115@gmail.com. Address: 599 Gwanak-ro Gwanak-gu, Department of Economics, Seoul National University, Seoul, 151-746, Republic of Korea; phone: 82-2-880-6345; fax: 82-2-886-4231.
I Introduction

Some private goods are publicly provided. Textbook microeconomic theory shows that the increasing returns-to-scale (henceforth, IRS) property gives rise to natural monopolies, and the regulation of natural monopolies is an important subject in applied economic analysis. Examples of the goods or industries with IRS properties include the utility, communications, and transportation which are highly regulated in most countries. In many instances, regulations of them take the form of public enterprises providing those private goods/services with IRS properties, i.e., public provision of private goods on the basis of the rationale that the competitive market principle of price equaling marginal cost (the $p = MC$ rule) does not allow operation of firms due to negative profits.$^1$

This paper argues that public enterprises provide private goods not only because of the IRS property, but also because they help relax the self-selection constraint of the optimal income tax problem. The intuition is that when some private goods (e.g., public transportation service) relative to other goods are more valuable to low-ability individuals than high-ability ones in terms of the marginal rate of substitution (henceforth, MRS), an appropriate pricing scheme can induce high-ability individuals to be discouraged to mimic low-ability ones, allowing for a room for Pareto improvement. In this case, our results provide a theoretical explanation for why public enterprises provide private goods at a low price for low-income individuals. In fact, the violation of the $p = MC$ rule implies that the production efficiency theorem by Diamond and Mirrlees (1971) does not hold. While this result appears surprising, the presence of IRS does not necessarily guarantee the production efficiency theorem by Diamond and Mirrlees (1971) to hold in our setting. In this particular environment, utilizing the information about consumption of the goods produced by public enterprises can offer the opportunities for more efficient redistribution.$^2$

$^1$An alternative to this marginal cost pricing is two-tier pricing system, charging some users a higher price while maintaining the price equaling marginal cost for other users. With this pricing system, profits on the high-price demanders compensate the losses incurred on the low-priced sales.

$^2$In a two-sector model with constant-returns-to-scale technologies in both sectors, Naito (1999) shows that, in addition to a non-linear income tax system, subsidizing the wages for the workers in public enterprises that employ a labor-intensive technology can enhance social welfare. This violates the production efficiency theorem but by inducing a factor price structure that is favorable to low-skilled labor the social welfare can improve.
There is a vast literature on optimal income taxation where efficient redistribution is the key issue in the presence of heterogeneity in earnings ability (see e.g., Mirrlees, 1971; Stiglitz, 1982). To our knowledge, however, there is little research addressing the interaction between optimal income tax structure and pricing of publicly provided private goods. We build upon the standard optimal income taxation models by Stiglitz (1982) and Boadway and Keen (1993), and then demonstrate the optimal tax and regulation policy combinations. In particular, we show the structure of nonlinear pricing of publicly provided private goods, and then how it interacts with optimal income taxation. At the least, this exercise broadens the horizon of optimal taxation and can shed light on the importance of policy mixes.

Another contribution of the paper comes from empirical application of our idea, which is rare in the optimal taxation literature. We propose an empirical framework that tests whether a particular good really helps relaxing the self-selection constrain.

The paper is organized as follows. Section II presents our models and optimization results. Section III describes the optimal pricing structure of the goods with IRS properties and its implications. Section IV shows empirical implementation of our theory, and Section V concludes with some discussions.

II The Model

Drawing on Stiglitz (1982) and Boadway and Keen (1993), we build a model with government in which optimal taxation and optimal regulation of public enterprises are both considered. In a model with income taxes, Stiglitz (1982) analyzes the set of Pareto efficient tax structure and formulates the canonical problem with self-selection. Later, Boadway and Keen (1993) consider a model with income taxes and public goods, and find that public good provision with optimal nonlinear taxes can deviate from the Samuelson Rule when two types of households are allowed to value a public good differently, using the self-selection approach. While sharing a similar spirit with the previous studies, we focus on optimal provision of private goods with the IRS property in the presence of another essential government instrument,

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In our paper, however, we use the information about a particular type of consumption that is favorable to low-income individuals for enhancing efficient redistribution.
income taxes. In this study, we explore optimal taxation with pricing of a publicly provided ‘private’ good, not a pure public good, which has not been studied yet to our knowledge.

### a Environment

Consider an economy with two types of households with different ability: low ability and high ability. Production is linear in labor supplies and individuals are paid according to their ability level: \( w_1 \) for the low ability and \( w_2 \) for the high ability. Type-\( i \) person with \( i = 1, 2 \) provides an amount of labor \( L_i \), and consumes \( X_i \) of a private good and \( Q_i \) of another private good with the IRS property provided by a government. The pre-tax income for type-\( i \) person is

\[
Y_i = w_i L_i. \tag{1}
\]

There are \( N_i \) households of type \( i \). Persons of type-\( i \) maximize their utility, given by strictly concave utility function \( U^i(X_i, L_i, Q_i) \). As in Boadway and Keen (1993), we allow the two types to value \( Q_i \) differently. Different tastes may arise from either different utility function or different abilities under identical utility. In the latter case, income differences will lead to different tastes. Including \( Q \) with \( Q_i \)’s being different across individuals is a new feature, compared to the traditional optimal income taxation literature. In addition to that, the prices of \( Q_i \) for each individuals are determined by a price function for privately provided good, denoted by \( p(Y_i) \). The social planner determines the structure of \( p(Y_i) \) in accordance with individuals’ income levels. The budget constraint of household \( i \) in the presence of the income tax is given by

\[
X_i + p(Y_i)Q_i \leq Y_i - T(Y_i), \tag{2}
\]

where \( T(Y_i) \) is a nonlinear tax function, \( p(Y_i) \) is a price of \( Q_i \) which implies the price relies on the type of a household and \( X_i \) is numeraire. Natural monopolies appear in the production of \( Q \), exhibiting IRS property and decreasing average costs over a broad range of output levels we consider. Marginal cost pricing of \( Q \) is, therefore, efficient but involves an operating loss which needs to be subsidized by the government.

The planner who chooses the function of nonlinear taxes is not able to
identify individuals by their ability, i.e., asymmetric information. That is, \( w_i \) and \( L_i \) are not observable but only information regarding \( Y_i \) is revealed. Hence, writing the utility function in terms of observable elements–\( Q_i, X_i \) and \( Y_i \)--is useful in self-selection problem where asymmetric information between households and the government exists. Following Stiglitz (1982), we posit type-\( i \) individual’s utility function as:

\[
U^i(X_i, L_i, Q_i) = U^i(X_i, Y_i/w_i, Q_i) \\
V^i(X_i, Y_i; Q_i)
\]  

where \( V^i_x = U^i_x, V^i_y = U^i_y/w_i \) and \( V^i_Q = U^i_Q \).

b Individual Problem

We first characterize the individual problem. The individual chooses \( \{X_i, Y_i, Q_i\} \) to maximize his or her utility \( V^i \) subject to his or her budget constraint. The Lagrangian expression for individual \( i \) can be written

\[
\mathcal{L} = V^i(X_i, Y_i, Q_i) + \lambda_i\{Y_i - T(Y_i) - X_i - p(Y_i)Q_i\},
\]

where \( \lambda_i \) is a Lagrange multiplier for the individual \( i \)'s budget constraint. We obtain the first-order conditions of \( X_i, Q_i \) and \( Y_i \):

\[
\begin{align*}
V^i_X &= \lambda_i, \\
V^i_Y &= -\lambda_i(1 - T' - p'Q), \\
V^i_Q &= \lambda_ip(Y_i),
\end{align*}
\]

where \( T' = \frac{\partial T(Y_i)}{\partial Y_i} \) and \( p' = \frac{\partial p(Y_i)}{\partial Y_i} \).

For later reference, we combine first-order conditions properly and have the following marginal rate of substitutions:

\[
\begin{align*}
\frac{V^i_y}{V^i_X} &= -(1 - T' - p'Q_i), \\
\frac{V^i_Q}{V^i_X} &= p(Y_i), \\
\frac{V^i_y}{V^i_Q} &= \frac{(1 - T' - p'Q_i)}{p(Y_i)}.
\end{align*}
\]
c Constraints

Before writing the planner’s problem, we conceptualize the problem and constraints. The social planner chooses the tax function $T(Y_i)$, price function $p(Y_i)$ for $Q$ and the total level of publicly provided private goods, $Q$ which becomes $N_1Q_1 + N_2Q_2$ by definition. The planner maximizes the objective function—the Pareto social welfare function—following the work of Boadway and Keen (1993). Precisely, we consider the problem of maximizing $V^1$ subject to a given level of $V^2$ denoted by $\overline{V^2}$. There are three constraints the planner faces: one is a revenue constraint and the other two are self-selection constraints. The revenue constraint reflects the government budget constraint:

$$\begin{align*}
AC(Q)Q &\leq \{p(Y_1)N_1Q_1 + p(Y_2)N_2Q_2\} \\
&\leq N_1\{Y_1 - X_1 - p(Y_1)Q_1\} + N_2\{Y_2 - X_2 - p(Y_2)Q_2\},
\end{align*}$$

where $AC(Q)$ is the average cost of producing $Q$. Equation (9) implies that the operating loss of the natural monopoly, i.e., total cost − revenue, should be subsidized by the tax revenue for continuation of production. Note that average cost of producing $Q$ is expected to decrease because of the IRS property, or economies of scale in production of $Q$. This revenue constraint, (9) can be simplified to:

$$C(Q) \leq N_1(Y_1 - X_1) + N_2(Y_2 - X_2), \quad (9')$$

where $C(Q)$ is total cost of producing $Q$. Self-selection constraints that make the households be at least well-off by consuming the consumption bundle meant for them are needed since persons are not identified by ability. Since there are two types, we need two self-selection constraints,

$$\begin{align*}
V^1(X_1, Q_1, Y_1) &\geq V^1(X_2, Q_2, Y_2), \\
V^2(X_2, Q_2, Y_2) &\geq V^2(X_1, Q_1, Y_1).
\end{align*}$$

(10)  (11)
Most of the studies in this literature, including Boadway and Keen (1993), however, assume that the first self-selection constraint is always satisfied since it is hard for a low-ability person to mimic a high-ability person. We will study with focus the ‘normal’ case where only the second self-selection constraint is binding.

### d Optimization Problem

The social planner maximizes $V^1$ subject to a given level of $V^2$ denoted as $\overline{V^2}$ under the two constraints explained above. Putting the elements together, the Lagrangian expression for the planner problem can be written as:

$$
\Omega(X_i, Y_i, Q_i, \mu, \xi, \gamma) = V^1(X_1, Y_1, Q_1) + \mu[V^2(X_2, Q_2, Y_2) - \overline{V^2}] + \xi[V^2(X_2, Q_2, Y_2) - V^2(X_1, Y_1, Q_1)] + \gamma[N_1(Y_1 - X_1) + N_2(Y_2 - X_2) - C(Q)], (12)
$$

where $\mu, \xi$ and $\gamma$ are Lagrange multipliers. Now, optimization of the problem yields the first-order conditions for $X_i, Y_i$ and $Q_i$:

$$
\begin{align*}
V^1_X - \xi \bar{V}^2_X - \gamma N_1 &= 0, \quad (13-1) \\
V^1_Y - \xi \bar{V}^2_Y + \gamma N_1 &= 0, \quad (13-2) \\
V^1_Q - \xi \bar{V}^2_Q - \gamma N_1 MC(Q) &= 0, \quad (13-3) \\
\mu V^2_X + \xi V^2_X - \gamma N_2 &= 0, \quad (13-4) \\
\mu V^2_Y + \xi V^2_Y + \gamma N_2 &= 0, \quad (13-5) \\
\mu V^2_Q + \xi V^2_Q - \gamma N_2 MC(Q) &= 0, \quad (13-6)
\end{align*}
$$

where $MC(Q) = \frac{\partial C(Q)}{\partial Q}$ and $\bar{V}^2 = V^2(X_1, Y_1, Q_1)$ representing the utility of the high-ability person when mimicking the low-ability one.

\[^2 \frac{\partial C(Q)}{\partial Q_i} = \frac{\partial C(Q)}{\partial Q} \frac{\partial Q}{\partial Q_i} \text{ and } \frac{\partial Q}{\partial Q_i} = N_i \text{ since } Q = N_1 Q_1 + N_2 Q_2.\]
We rearrange these first-order conditions for later reference as follows:

\[
\begin{align*}
V_Y^1 &= -\frac{\xi V_Y^2 + \gamma N_1}{\xi V_X^2 + \gamma N_1}, \\
V_X^1 &= \frac{\xi V_X^2 + \gamma N_1}{\xi V_X^2 + \gamma N_1}, \\
V_Q^1 &= \frac{\xi V_Q^2 + \gamma N_1 MC(Q)}{\xi V_X^2 + \gamma N_1}, \\
V_Y^1 &= \frac{\xi V_Y^2 + \gamma N_1}{\xi V_X^2 + \gamma N_1}, \\
V_Q^1 &= \frac{\xi V_Q^2 + \gamma N_1 MC(Q)}{\xi V_X^2 + \gamma N_1}, \\
V_Y^2 &= -1, \\
V_X^2 &= \frac{\xi V_Q^2 + \gamma N_1 MC(Q)}{\xi V_X^2 + \gamma N_1}, \\
V_Q^2 &= \frac{1}{MC(Q)}.
\end{align*}
\] (14-19)

Analyzing these conditions yields the conventional optimal income taxation results of Stiglitz (1982) and when combined with the optimal pricing conditions for \(Q_i\), they yield some new results. To be concrete, equations (17) and (6) yield \(T' + p'Q_i = 0\) for the high-ability individuals, which does not necessarily imply that \(T' = 0\) and \(p' = 0\). We will discuss this issue in a later subsection. From (14) and (6), one can deduce that \(T' + p'Q_i > 0\) for the low-ability individuals, which is greater than that of the high-ability ones. To see this, following Stiglitz (1982), define

\[
\begin{align*}
\alpha^i &= -\frac{\partial V^i/\partial Y_1}{\partial V^i/\partial X_1}, \\
\nu &= \frac{\xi \partial V^2/\partial X_1}{\gamma N_1}.
\end{align*}
\] (20-21)

Then, (14) can be written as:
\[- \frac{V_1^1}{V_1^X} = \alpha^1 \]
\[= \frac{1 - \xi(\partial V^2/\partial Y_1)/\gamma N_1}{1 + \xi(\partial V^2/\partial X_1)/\gamma N_1} \]
\[= \frac{1 + \nu \alpha^2}{1 + \nu} \]
\[= \alpha^2 + \frac{1 - \alpha^2}{1 + \nu}. \]

By invoking the "well-known" single crossing property, \( \alpha^1 > \alpha^2 \), we can obtain:

\[\alpha^2 < \alpha^1 < 1. \tag{26}\]

From this, we can prove the positive marginal tax rate on low-ability individuals as follows:

\[\alpha^1 = 1 - T' - p'Q_1 \text{ (from (6))} \tag{27}\]
\[< 1, \tag{28}\]
which implies \( T' + p'Q_1 > 0. \)

As (16) is redundant with (14) and (15), we can leave out (16). In the same way, (19) can be omitted for convenience. Therefore, using (15) and (18), we can discuss the optimal pricing of \( Q_i \) below.

### III Optimal Pricing

Now, we can discuss optimal pricing of the publicly provided private good, \( Q \) by combining (15) and (7), and (18) and (7), respectively. First, combining (18) and (7), we obtain:

\[MC(Q) = p(Y_2), \tag{29}\]
which is a well-known standard result for efficiency of \( p = MC \). Charging the marginal cost to high-ability individuals is optimal. Next, combining
(15) and (7) in a similar way yields

\[
\frac{\xi \hat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi \hat{V}_X^2 + \gamma N_1} = p(Y_1),
\]

which looks complex but contains a key implication of the paper.

a Two interesting cases

To interpret (30), we first need to analyze (15), the \(MRS_{QX}\) of type 1 by considering two cases:

(a) \(\frac{\hat{V}_Q^2}{V_Q^2} = \frac{V_Q^1}{V_Q^1}, \) or \(MRS_{QX}^2 = MRS_{QX}^1,\)

(b) \(\frac{\hat{V}_Q^2}{V_Q^2} < \frac{V_Q^1}{V_Q^1}, \) or \(MRS_{QX}^2 < MRS_{QX}^1,\)

where the latter case, (b) is more interesting for our study because some goods are more preferred by low-income individuals in a relative sense.\(^4\)

The case of equal MRS. First, we start with the case (a) and have

\[
\frac{V_Q^1}{V_Q^1} = \frac{\xi \hat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi \hat{V}_X^2 + \gamma N_1} \quad \text{(from (15))} \quad (31)
\]

\[
= \frac{\hat{V}_Q^2}{V_Q^2} \quad \text{(case a)},
\]

where the first line of (31) is equal to (15), and the second line is simply the case (a). Therefore, we obtain

\[
\hat{V}_Q^2/\hat{V}_X^2 = \{\xi \hat{V}_Q^2 + \gamma N_1 MC(Q)\}/\{\xi \hat{V}_X^2 + \gamma N_1\},
\]

which is simplified to

\(^3\)\(MRS_{QX}^2 = MRS_{QX}^1\) means the consumption goods \(X_i\) and \(Q_i\) are weakly separable from \(L\) in the utility of the households. This happens when both a mimicker and a low-ability person have the same valuation about \(X_i\) and \(Q_i\) regardless of \(L\). Given that some publicly provided private goods are not irrelevant to labor supply or intrinsically more valuable to low-income individuals, the preference restriction of \(MRS_{QX}^2 = MRS_{QX}^1\) is unrealistic in our context.

\(^4\)This is type of relative comparison typically arises in discussions of international trade issues in a Heckman-Ohlin model.
\( \hat{V}_Q^2 / \hat{V}_X^2 = MC(Q) \), or \( \hat{V}_Q^2 / \hat{V}_X^2 = V_Q^1 / V_X^1 = MC(Q) \). With this and an optimizing process equating \( MRS_{QX}^1 \) to the price ratio, we obtain a conventional result, \( MC(Q) = p(Y_1) \) and \( MC(Q) = p(Y_2) \) from (20). It implies that the \( p = MC \) rule still holds under the case (a) of \( MRS_{QX}^1 = MRS_{QX}^1 \).

The realistic case. Next, we consider the case (b) which is more realistic for our case. Through a similar analysis, we obtain

\[
\frac{V_Q^1}{V_X^1} = \frac{\xi\hat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi\hat{V}_X^2 + \gamma N_1} \quad \text{(from (15))} \tag{32}
\]

\[\frac{\hat{V}_Q^2}{\hat{V}_X^2} \quad \text{(case b)},\]

where the first line of (32) is equal to (15), and the second line is simply the case (b). Therefore, we obtain

\[ \hat{V}_Q^2 / \hat{V}_X^2 < \frac{\xi\hat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi\hat{V}_X^2 + \gamma N_1}, \] which is simplified to

\[ \hat{V}_Q^2 / \hat{V}_X^2 < MC(Q). \]

Substituting \( \hat{V}_Q^2 < \hat{V}_X^2 MC(Q) \) into \( \hat{V}_Q^2 \) in \( \{ \xi\hat{V}_Q^2 + \gamma N_1 MC(Q) \} / \{ \xi\hat{V}_X^2 + \gamma N_1 \} \) yields:

\[
\frac{\xi\hat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi\hat{V}_X^2 + \gamma N_1} < \frac{\xi\hat{V}_Q^2 MC(Q) + \gamma N_1 MC(Q)}{\xi\hat{V}_X^2 + \gamma N_1} \tag{33}
\]

\[= MC(Q). \]

Given

\[
\frac{V_Q^1}{V_X^1} = \frac{\xi\hat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi\hat{V}_X^2 + \gamma N_1}, \tag{34}
\]

we obtain the result:

\[
\frac{V_Q^1}{V_X^1} < MC(Q). \tag{35}
\]
Figure 1: Note that $Q^* \equiv Q_1^* N_1 + Q_2^* N_1$ and $p_1$ is lower than $p_2$. The planner’s solution is where $p_2 = MC(Q^*)$ holds. Tax revenue partially covers total production cost.

The last result, $\frac{V_1}{V_X} < MC(Q)$, arises when the marginal evaluation to the mimicker is less than that to the low-ability person: $\overline{MRS}^2_{QX} < MRS^1_{QX}$. As optimizing households equate $MRS_{QX}$ to the price ratio, we obtain:

\[
\begin{align*}
    p(Y_1) &< MC(Q), \\
    p(Y_2) &= MC(Q) \quad \text{(from (20)),}
\end{align*}
\]

which means $p(Y_1) < p(Y_2)$. The deviation from the standard result, $p = MC$ and $MRS^i_{QX} = MRS^j_{QX}$ is for “efficient redistribution” in the presence of a private good with the IRS property and $\overline{MRS}^2_{QX} < MRS^1_{QX}$. Note that the private good with a IRS property here is comparatively more useful for low-income individuals, and this property has been utilized when setting the optimal pricing for $Q_1 < Q_2$. Figure 1 describes the equilibrium for case (b). The diagram (a) depicts aggregate demand curve of the low-ability persons, (b) depicts that of the high-ability ones and (c) contains marginal cost and average cost curve for $Q$. In equilibrium, the price for the high-ability equals to $MC(Q)$ and that of the low-ability is lower than $MC(Q)$.

We summarize the discussion above as follows:
Proposition 1  In the presence of optimal nonlinear income taxation, the rule for optimal provision of publicly provided private goods with the IRS property involves the pricing $p(Y_1) = MC(Q)$ as the marginal evaluation of the goods to the mimicker is less than, equal to, or greater than that to the low-ability person (i.e., $MR^2_{QX} <, =, > MR^1_{QX}$).

Interpretation. An intuitive interpretation applies to the optimal taxation and pricing result, explained above. To be concrete, consider the case in which the low-ability person values the publicly provided private good more than the mimicker, so $MR^2_{QX} < MR^1_{QX}$. Here, Proposition 1 tells that at the planner’s optimum, $p(Y_1) < MC(Q)$ holds so the good should be "over-consumed" by the low-ability person in contrast to the traditional rule. For this to do so, it should be possible to show that a Pareto improvement is possible starting at a traditional optimal income tax equilibrium with the $p = MC(Q)$ rule satisfied. To see this is possible indeed, consider the following thought experiment. We start at $p(Y_1) = p(Y_2) = MC(Q)$, or $MR^1_{QX} = MR^2_{QX} = MC(Q)$, where an efficient public provision of private good has been believed to occur in accordance with the literature. Next, imagine increasing $Q_i$ with $i = 1, 2$ incrementally and adjusting income tax structure such that each person pays as much as his or her $MR_{QX}$ for the additional $Q_i$. There will be no change in the welfare of either person 1 or 2 and government budget will not have changed. However, since the additional $Q_i$ costs a mimicking person $MR^1_{QX}$ which is greater than the mimicker’s valuation, $MR^2_{QX}$, the mimicker will be worse off. That is, the self-selection constraint is relaxed (i.e., mimicking is less attractive) and a change in the optimal tax structure and pricing function of $Q$ can be undertaken which will make both persons better off. That is, $T' + p'$ for person 1 can be lowered with the same total tax revenue being collected. Person 1 is made better off, without inducing person 2 to mimic. At the same time, the higher preference for $Q$ by low-ability individuals allows for $p(Y_1) < p(Y_2)$ for efficient redistribution.

b  Unobservability of consumption

The previous section is based on the presumption that the government is able to observe individuals’ consumption including $Q$. In some instances,
however, consumption of \( Q \) may be unobservable due to lack of information or difficulties of collecting information about consumption. It is interesting to examine whether unobservability of \( Q \) makes a difference to our earlier results.

**Proposition 2**  *In the absence of information about consumption of \( Q \), the rule for optimal provision of publicly provided private goods with the IRS property involves the uniform pricing, \( p(Y_1) = p(Y_2) = MC(Q) \). See the proof at the appendix.*

The intuition behind Proposition 2 is that since \( Q \) does not play the role of relaxing the self-selection constraint, the efficiency gain from non-uniform pricing no longer exists, which justifies the uniform pricing.

c  **Complementarity between \( Q \) and \( L \)**

Often public provision of private goods is justified when those goods help labor supply, i.e., complementarity between \( Q \) and \( L \). This subsection will discuss how the complementarity affects our results about optimal income tax and pricing of \( Q, T' + p'Q_i \). Complementarity of \( Q \) to labor – marginal willingness to pay in terms of \( MRS_{Q,X} \) rises with labor – implies that differentiating \( MRS_{Q,X}^i \) from (5-1) and (5-3) with respect to labor yields a positive sign:

\[
\frac{\partial MRS_{Q,X}^i}{\partial L_i} = p'_i w_i \quad \text{(differentiating } MRS_{Q,X}^i \text{ w.r.t labor)} \quad (38)
\]

\[
> 0, \quad \text{(complementarity between } Q \text{ and } L)\]

where \( p'_i = \partial p(Y_i)/\partial Y_i \), and \( w_i \) is non-negative. From this, one can deduce that \( p_t \) should be positive when \( Q \) is complementary to labor. Given \( T' + p'Q_i = 0 \) for the high-ability person due to (17) combined with (6), the marginal income tax rate of the high-ability person is negative in this case. Similarly, since \( p_t \) is also positive for low-ability individuals, \( T' \) can go down further leading to a greater supply of labor, i.e., less distortion in work. Therefore, we can find that the complementarity reinforces our earlier results with greater work incentives. This can be seen as an application of Corlette and Hague (1953): since \( Q \) is complementary to
labor supply, favoring $Q$ through offering a lower price or a better tax treatment is justified. As we specified the preference in the previous section, if the preference difference arises from different earnings ability and hence different labor supply, the complementarity of $Q$ to labor occurs which leads to greater incentives to work to both low and high ability individuals.

On the other hand, if $Q$ is independent of labor, we obtain:

$$\frac{\partial MRS_i^Q X}{\partial L_i} = p_i^\prime w_i = 0,$$

where $p_i^\prime = \partial p(Y_i)/\partial Y_i$, and $w_i$ is non-negative. From this, one can deduce that $p_i^\prime$ should be zero which leads to that the marginal income tax rate of the high-ability individuals is zero, as in the literature. In this case, other things being constant, $T^\prime$ goes up, compared to the complementarity case, leading to less work. In a similar way, the marginal income tax rate of low-ability individuals remains positive which discourages their labor supply more compared to the case above.

**Proposition 3** With a complementarity between $Q$ and $L$, $p_i^\prime$ is positive, $T^\prime$ is negative for high-ability individuals, and $T^\prime$ gets smaller for low-ability individuals, boosting work incentives for all individuals.

**d Discussion**

Using Propositions 1 and 2, we can offer a more specific explanation about the public provision of a private good, given the good is complementary to labor and more preferred by the low-ability individuals. Proposition 1 shows that an optimal pricing of $Q$ is “discrimitory” with $p(Y_1) < MC(Q) = p(Y_2)$ when the low-ability individuals’ valuation of $Q$ is higher than the high-ability ones’ valuation. Furthermore, this difference in preferences generates an Pareto improvement—represented by a lowered value of $T^\prime + p_i^\prime Q_i$ for the
low-ability ones—at the traditional optimal income tax equilibrium with the \( p(Y_1) = p(Y_2) = MC(Q) \) rule satisfied. This implies the possibility of reduced marginal income tax rate of the low-ability individuals, when \( p' \) is fixed. From Proposition 2, we can deduct the rule for determining \( p_i \) and it relies on the complementarity of \( Q \) to labor, which, for example, makes \( p_i \) positive as \( Q \) is complementary to labor. Taking advantage of these results, we can conclude that given the characteristics of \( Q \)—complementary to labor and more valuable to the low-ability individuals, providing \( Q \) at a lower price to the low-ability persons makes them better off with a reduced marginal income tax rate, which eventually leads to greater labor supply. Even with these benefits that the low-ability individuals newly enjoy—the lower price and marginal income tax rate, the high-ability individuals are not willing to mimic since \( Q \) is less valuable to them. This is another interpretation of relaxation of the self-selection constraint. Resulting welfare improvement occurs when there is a difference in preferences for \( Q \) and a government provides that good.

In fact, some countries implement subsidy programs on the consumption of electricity, public transportation, etc., based on the income of households. Promising is empirical investigation of whether those publicly-provided private goods exhibit an IRS feature, and whether consumption of those goods really favor low-income individuals, which is the topic of the next section. Finally, it should also be noted that implementation of a non-linear pricing policy requires observability of consumption of \( Q \). Otherwise, the usual uniform \( p = MC \) rule for all individuals is still the social-welfare maximizing result.

### IV Empirical Results

#### a Identification idea

So far we have dealt with only theoretical analysis of public provision of private goods with IRS property when there exists preference difference among different types of individuals. In this section, we examine whether our idea can apply to the data.

In our paper, preference difference is defined in terms of \( MRS_{QX}^i \). Especially, when low-ability person values good \( Q \) more than high-ability one, the following inequality holds:
where $\overline{MRS}^2_{QX}$ denotes marginal rate of substitution for mimickers and $\overline{MRS}^1_{QX}$ denotes that of low-ability ones. The figure below depicts this case.

Figure 2: Note that at point A where both a mimicker and a low-ability individual consume the same Q and X, the MRS$_{QX}$ is steeper for the mimicker. Therefore, high-ability individuals self-select the consumption bundle B.

The diagram shows that mimickers are eager to choose point B rather than point A with the same income. This is because high-ability persons value Q less than the low-ability ones. Putting it in other way, we can say high-ability individuals tend to spend a small share of total expenditure on Q, by choosing point B while the low-ability individuals are willing to spend a larger portion of their income on Q by choosing point A.

As we cannot identify type of each individual in a real world, we need to use proxy for ability such as level of education or labor hours given income. As the education level is more direct indication of level of ability without loss of generality, our initial specification will utilize level of education as proxy for ability, and next labor hour.
b Empirical specification

Our empirical strategy is to find evidence that the share of expenditure on Q falls with education, holding income constant. According to the figure 2, the person with high ability spends a less share of expenditure on Q when we control for income. The dependent variable is the share of total household expenditure on Q denoted by W and we posit Specification 1 as follows:

**Specification 1**

\[ W_i = \alpha + \beta_1 \log(m_i) + \beta_2 DU NI_i + Z'\eta \quad (40) \]

Here, \( W_i \) is the household \( i \)'s share of expenditure on the good of interest Q, \( m_i \) is the household’s total expenditure which alternatively measures permanent income, \( DU NI_i \) is the dummy variable for receiving college education for the household head and \( Z \) is a vector of other characteristics of the household which affect consumption of Q. A change in share of expenditure on Q under fixed income can be identified by the coefficient of \( DU NI_i \), \( \beta_2 \) whose sign will be negative if low-ability individuals prefer Q more than the high-ability individuals.

Next, let us consider another specification of the model, using labor hour given income as proxy for ability since it is apparent that the person who is more able works less labor hour to earn the same amount of income. Thus, referring to the figure 2, we can say that the person supplying less labor to earn the same income is the high-ability one who spends a smaller share of expenditure on Q by choosing point B. Our empirical strategy here is to check whether those who work longer hours have a greater share of expenditure on Q, holding income constant. Now, we posit Specification 2 as follows:

**Specification 2.**

\[ W_i = \alpha + \delta_1 \log(m_i) + \delta_2 \log(L_i) + \delta_3 \log(L_i) \log(m_i) + \delta_4 DU NI_i + Z'\eta, \quad (41) \]

where, \( L_i \) is labor hour of the head of household \( i \). A change in share of expenditure on the good under fixed income is now identified by \( \delta_2 \) which will be positive if the low-ability individuals prefer Q more than the high-ability ones.
c Data

Our data were taken from the KLIPS (Korean Labor Income Panel Study). We included all households observed in 2007. Description of variables and statistics are given in Table A and the later tables. According to the description of the statistics in Table A, most of the household heads are male and their average age is 50.73. Most households are small, consisting of 2.51 members and 1.15 children on average. The average of log of weekly labor hour is 3.86 and 44% of the household heads have received 2-year college or more years of education.

d Empirical results

With this data set and variables, we have obtained following result from regression in Table B. The overall result of the initial regression shows that the estimated coefficient for level of education, $\beta_2$ is negative, which indicates individuals with lower education level, the low-ability ones, tend to consume more $Q$. Focusing on the results in column (4) of Table A, the estimated value of $\beta_2$ is $-0.0028$ with statistical significance. The last column of Table B involving the most variables reports $-0.0012$ for the estimate of $\beta_2$, whose estimate, however, is statistically insignificant.

Next, the equivalent regression result is displayed in Table C. According to the second regression result in Table C, the estimated coefficient on labor hour is positive. Focusing on the result in column (5) of Table C, the estimate of $\delta_2$ is 0.0580 with p-value less than 0.01. We have tried to control for the level of income up to quadratic terms to capture nonlinear effects. To be more precise, looking at $\delta_2$ only does not fully capture the effect of labor supply on the share $W$. This is because an incentive in labor supply brings an increase in income as follows:

$$\frac{\partial W_i}{\partial \log(L_i)} \bigg|_{m_i} = \delta_2 + \delta_3 \log(m_i) - \underbrace{\delta_1 \frac{\partial \log(m_i)}{\partial \log L_i}}_{\text{holding income effect constant}}$$

This formulation shows the true impact of an increase in $L_i$ on $W_i$, holding income constant.\footnote{This is equivalent to separating substitution effect from income effect in analysis of price effect.} In this model, we expect $\frac{\partial W_i}{\partial \log(L_i)} \bigg|_{m_i}$ to be positive since individuals with higher labor hours given income — low-ability individuals...
in our context – tend to spend a larger fraction of income on $Q$. Combining
the equation (42) and regression result in the last column of Table C yields

$$\frac{\partial W_i}{\partial \log(L_i)} \approx 7.23$$

(43)

where we made use of $\frac{\partial \log m}{\partial \log L} = 0.1992$ from regression of $\log h$ on $\log m$.

This result supports the case of higher preference of the low-ability
individuals for $Q$ such as public transportation service.

Although not shown for brevity, we find that other equivalent regressions
for corresponding goods such as telecommunication and housing utility give
qualitatively similar results according to the regression results given in Table
D. We find telecommunication and housing utility are more preferred by the
low-ability individuals since the estimated coefficient for the level of house-
hold head’s education is negative with high statistical significance for the
both cases. In addition, we find an increase in the share of total expenditure
on telecommunication when labor supply increases, as we see the positive
sign of coefficient on the labor hour, 0.0627 with statistical significance.

V Summary and Conclusions

We have studied the policy mix of optimal income taxation and pricing of
publicly provided private goods with the IRS property. Our main results
have established that public enterprises provide private goods not only for
the IRS property, but also for relaxing the incentive problem of the tax
system: they help relax the self-selection constraint of the optimal income
tax problem through nonlinear pricing.

The intuition behind the results is that when some private goods with
IRS properties (e.g., public transportation) relative to other goods are more
valuable to low-ability individuals than high-ability ones in terms of the mar-
ginal rate of substitution (MRS), high-ability individuals are discouraged to
mimic low-ability ones. In this particular case, our results provided theoreti-
cal underpinning for the low price of publicly provided goods for low-income
individuals. Owing to the nonlinear pricing, the optimal income tax rate for
low-income individuals can go down, allowing for welfare improvement. We
believe that the highlighted policy mix between optimal income tax struc-
ture and pricing of publicly provided private goods can offer new insight into optimal tax and expenditure policy combinations.

Appendix

Proof of Proposition 2 In the previous section, we have assumed a government can obtain information of each individuals’ consumption, which is not available in reality. As the government cannot observe consumption of individuals, the only information the government uses to discern type is the pre-tax income, \( Y_i \). That is, a mimicker does not have to consume as much as \( X_1 \) and \( Q_1 \) any more but adjust his or her labor hours to obtain the pre-tax income, \( Y_1 \). In this case, the mimicker is able to choose amount of \( X \) and \( Q \) according to its preference under the given budget constraint

\[
Y_1 - T(Y_1) = \tilde{X}_2(Y_1) - P(Y_1)\tilde{Q}_2(Y_1),
\]

where \( \tilde{X}^2 \) and \( \tilde{Q}^2 \) represent choices of \( X \) and \( Q \) of the high-ability individuals, when mimicking the low-ability ones with pre-tax income, income tax and price of \( Q \) given as \( Y_1, T(Y_1) \) and \( P(Y_1) \). Note that \( \tilde{X}^2 \) and \( \tilde{Q}^2 \) should be the optimal function of \( Y_1 \) as the mimicker’s choices of \( X \) and \( Q \) now depends on the income of low-ability individuals. We can rewrite the mimicker’s optimality problem with the modified budget constraint as follows:

\[
L' = V^2(\tilde{X}_2(Y_1), \tilde{Q}_2(Y_1), Y_1) + \lambda_2^f \{ Y_1 - T(Y_1) - \tilde{X}_2(Y_1) - P(Y_1)\tilde{Q}_2(Y_1) \},
\]

where \( \lambda_2^f \) is the Lagrangian multiplier in the case of unobservability of \( Q \). We can derive first order conditions for \( \tilde{X}_2 \) and \( \tilde{Q}_2 \) which simply become

\[
\begin{align*}
\tilde{V}_\tilde{X}^2 - \lambda_2^f &= 0, \\
\tilde{V}_\tilde{Q}^2 - P(Y_1)\lambda_2^f &= 0,
\end{align*}
\]

where \( \tilde{V}_\tilde{X}^2 = \frac{\partial V^2}{\partial \tilde{X}_2} \) and \( \tilde{V}_\tilde{Q}^2 = \frac{\partial V^2}{\partial \tilde{Q}_2} \). With this newly defined optimal problem
of mimicker, the social planner has to change its self-selection constraint (10) into

\[ V^2(X_2, Q_2, Y_2) \geq V^2(\tilde{X}_2(Y_1), \tilde{Q}_2(Y_1), Y_1), \]

and the Lagrangian expression for the social planner should be

\[ \Omega'(X_i, Y_i, Q_i, \mu, \xi, \gamma) = V^1(X_1, Y_1, Q_1) + \mu[V^2(X_2, Q_2, Y_2) - V^2] \\
+ \xi[V^2(X_2, Y_2, Q_2) - V^2(\tilde{X}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))] \\
+ \gamma[N_1(Y_1 - X_1) + N_2(Y_2 - X_2) - C(Q)], \]

where \( \mu, \xi \) and \( \gamma \) are Lagrangian multipliers. With this, one can derive another first order conditions on \( X_i, Y_i \) and \( Q_i \)

\[ V^1_X - \gamma N_1 = 0, \quad (50-1) \]
\[ V^1_Y = -\xi V^2(\tilde{X}_2(Y_1), Y_1, \tilde{Q}_2(Y_1)) + \gamma N_1 = 0, \quad (50-2) \]
\[ V^1_Q = \mu V^2 + \xi V^2 - \gamma N_2 = 0, \quad (50-3) \]
\[ \mu V^2_X + \xi V^2_X + \gamma N_2 = 0, \quad (50-4) \]
\[ \mu V^2_Q + \xi V^2_Q - \gamma N_2 MC(Q) = 0, \quad (50-5) \]
\[ \mu V^2_Y + \xi V^2_Y - \gamma N_2 MC(Q) = 0, \quad (50-6) \]

where \( MC(Q) = \frac{\partial C(Q)}{\partial Q} \). As there is no change in optimal conditions for the high-ability individuals, we will now restrict attention to equation (50-1), (50-2) and (50-3). Rearranging the first order conditions of low-ability ones, we obtain

\[ \frac{V^1_V}{V^1_X} = -\xi \frac{dV^2(\tilde{X}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))}{dY_1} + \gamma N_1, \quad (51) \]
\[ \frac{V^1_Q}{V^1_X} = MC(Q), \quad (52) \]
\[ \frac{V^1_Y}{V^1_Q} = -\xi \frac{dV^2(\tilde{X}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))}{dY_1} + \gamma N_1 MC(Q) \quad (53) \]
Analyzing these conditions yields different results of optimal pricing of $Q$ and income taxation. To be concrete, equation (52) and (7) yield $p(Y_1) = MC(Q)$, which implies the uniform price for good $Q$ regardless of difference in preference of each types. The intuition is that since $Q$ does not play the role of relaxing the self-selection constraint, the efficiency gain from non-uniform pricing no longer exists, which justifies this uniform pricing. The optimal taxation for low-ability individuals is derived from equation (51) and (6), which can be written as follows:

$$
\frac{V^1_Y}{V^1_X} = \frac{-\xi \frac{dV^2(\tilde{x}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))}{dY_1} + \gamma N_1}{\gamma N_1} \quad \text{(from equation (50))}
$$

$$
= -(1 - \frac{\xi}{\gamma N_1} \frac{\partial V^2(\tilde{x}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))}{\partial Y_1})
$$

$$
= -(1 - T' - p'Q_1) \quad \text{(from equation (6)), (54)}
$$

where the first line of (54) is equal to equation (51), the second line simply a rearrangement of equation (51) and the last line equation (6). From (51) one can deduce $T' + p'Q_1 = \frac{\xi}{\gamma N_1} \frac{dV^2(\tilde{x}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))}{dY_1}$ where $\frac{\xi}{\gamma N_1} > 0$ but the sign of $\frac{\partial V^2(\tilde{x}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))}{\partial Y_1}$ is unclear. For analytic purpose, we need to specify $\frac{dV^2(\tilde{x}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))}{dY_1}$ and we obtain

$$
\frac{dV^2(\tilde{x}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))}{dY_1} = \frac{\partial V^2 d\tilde{x}_2}{\partial \tilde{x}_2 dY_1} + \frac{\partial V^2 d\tilde{Q}_2}{\partial \tilde{Q}_2 dY_1} + \frac{\partial V^2}{\partial Y_1}. \quad (55)
$$

Using equation (46) and (47), and definition of Lagragian multiplier, we can rewrite and rearrange equation (55) as follows:

$$
\frac{\partial V^2(\tilde{x}_2(Y_1), Y_1, \tilde{Q}_2(Y_1))}{\partial Y_1} = \lambda_2P(Y_1)\frac{\partial \tilde{x}_2}{\partial Y_1} + P(Y_1)\frac{\partial \tilde{Q}_2}{\partial Y_1} + \lambda_2'
$$

$$
= \lambda_2' + P(Y_1)\frac{\partial \tilde{Q}_2}{\partial Y_1} + 1 \quad (56)
$$

where $\lambda_2'$ and $\frac{\partial \tilde{x}_2}{\partial Y_1}$ are expected to be positive while $\frac{\partial \tilde{Q}_2}{\partial Y_1}$ is possibly negative since publicly provided public good $Q$ is usually favorable for low-income individuals and has negative income elasticity. Following the definition of income elasticity, we can rewrite the last line of equation (56)
as

\[
\frac{\partial V^2(\widetilde{X}_2(Y_1), Y_1, \widetilde{Q}_2(Y_1))}{\partial Y_1} = \lambda_2[\frac{\partial \widetilde{X}_2}{\partial Y_1} + P(Y_1) \frac{\partial \widetilde{Q}_2}{\partial Y_1} + 1] \quad (57)
\]

\[
= \lambda_2[\epsilon_{X,Y} \frac{Y_1}{X_2} + \epsilon_{Q,Y} P(Y_1) \frac{Y_1}{Q_2} + 1] \quad (58)
\]

where \( \epsilon_{X,Y} \) and \( \epsilon_{Q,Y} \) are income elasticities of demand for \( Q \) and \( X \) respectively. Without loss of generality, we can assume that consumption of \( X \) and \( Q \) rises with increase in income, equivalently \( X \) and \( Q \) are normal goods. This leads to \( \epsilon_{X,Y} > 0 \) and \( \epsilon_{Q,Y} > 0 \). In this case, as equation (58) leads to \( \frac{\partial V^2(\widetilde{X}_2(Y_1), Y_1, \widetilde{Q}_2(Y_1))}{\partial Y_1} > 0 \) and eventually

\[
\frac{\epsilon}{\gamma N_1} \frac{\partial \epsilon^2(\widetilde{X}_2(Y_1), Y_1, \widetilde{Q}_2(Y_1))}{\partial Y_1} = T' + p'Q_1 > 0 \] holds, we can conclude that the marginal tax rate combined with the pricing condition has to be positive for the low-ability individuals, i.e., the same result of the previous observable case.

**References**


Table A: Descriptive statistics for the KLIPS data

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<th>Variables</th>
<th>Mean</th>
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<th>Max.</th>
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<td>0.03</td>
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<td>DCAR</td>
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<td>DOWNER</td>
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<td>28.60</td>
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</table>

The variables included from the KLIPS are:

*Share of public transportation.* Share of expenditure on public transportation is rated as \( W = \frac{\text{Expenditure on public transportation}}{\text{Total expenditure}} \). Expenditure. Permanent income is proxied by total monthly expenditure of the household excluding monthly rent to control for the limitation of measuring imputed rents.

*Labor hour.* It is weekly hours of work including both the self-employed and employed. \( LHOUR \) is log of weekly labor hours.

*Household characteristics.* Household head characteristics can provide proper alternatives to measuring households’ characteristics affecting consumption pattern.

AGE: Age of household head. AGE2, AGE3 and AGE4 represent square, cubic and quadratic terms of AGE, respectively,
SEX: Gender of household head, (SEX=1 if a household head is female)
DUNI: Dummy variable for household head receiving 2-year college or more years of education (DUN=1 if a household head has graduated from college or university).

*Household composition.* Household size affects the consumption level of the household even though we can ignore it in our context. However, as particular members in households also have effect on consumption on public transportation, we need to include the number of people in the household of particular types as variables:

- NMEM: The number of household members aged over 15,
- NCHILD: The number of children.

*Region.* As area households resides may have impact on consumption pattern, we control for the location roughly.

- DSEOUL: Dummy variable for dwelling in Seoul, the capital city of Korea (DSEOUL=1, if residents of Seoul),
- DGWANG: Dummy variable for dwelling in 6 Gwangyeok cities, which is compatible to Seoul in size and other political importance (DGWANG=1, if residing in Gwangyeok cities).

*Other household characteristics.* Other than household composition, asset information such as owning a house and/or a car may affect the consumption of public transportation. Therefore, we include followings,

- DCAR: Dummy variable for owning a car (DCAR=1, if a household has its own car),
- DOWNER: Dummy variable for owning a house (DOWNER=1, if a household owns its housing).

---

\[ \frac{\text{Expenditure on certain good}}{\text{Total expenditure}} \]

Share of expenditure on a good is defined as \( W = \frac{\text{Expenditure on certain good}}{\text{Total expenditure}} \). If we are to consider the effect of household size, both the total expenditure and expenditure on a certain good have to be deflated by the number of household members which leads to cancellation of the effect, and \( W \) eventually remains the same as before. This is the reason why we ignore the size of household even though it seems critical.
Table B. Regression Results: Specification 1

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<td>1.2681**</td>
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Obs.  4768  
R²  0.1583  0.2133  0.1666  0.1851  0.2236  
F-statistics  89.44  49.95  79.24  77.11  85.50

* 5%  
** 1%  

Figures in parantheses denotes standard error.
Table C. Regression Results: Specification 2

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Obs. 2223
R² 0.1465 0.2133 0.2212 0.2498 0.2903
F-statistics 63.41 49.95 44.81 45.91 50.10

Figures in parantheses denotes standard error.
*:*5%
**:*1%
Table D. Regression Results

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Observation 2147  2147
R² 0.3311  0.2284
F-statistics 58.53  35.00

Figures in parantheses denotes standard error.

*:5%
**:1%