Municipal Merger under Tax Competition with Debt

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Abstract

This paper analyzes whether municipal merger increases social welfare in a dynamic tax competition model. Though some papers about tax competition show that municipal merger increases social welfare because it terminates tax competition, the analysis finds that a welfare-decrease effect of municipal merger might eclipse a welfare-increase effect of municipal merger since it causes ‘common pool problem’. Another result of this paper shows that social welfare can be recovered employing the policies which reduce the effect from the common pool problem.

Keywords: Municipal merger; Common pool; Tax competition

JEL Classification: H71; H72; H74

1 Introduction

In many countries, municipal mergers have been implemented to improve social welfare. In these countries, the central governments try to implement municipal merger and some of them even compel municipalities to get merged*1, while they usually take some measures to minimize the negative impact from merger when mergers are implemented. Then, do municipal mergers really increase social welfare? And, if not, what kinds of policies are useful to relax the negative impact from merger?

There are many articles trying to answer the first question and they try to explain how mergers affect social welfare. Among them, there are two different discussions about social welfare and municipal mergers. One focuses on internalization of externalities caused by municipal mergers. When municipalities make externalities, internalizing them will lead to welfare improvement. There are large number of articles imply that various externalities are made by local governments, and, in particular, inter-governmental competitions such as tax competition are examined a lot. Considering this, municipal mergers will terminate competition between municipalities which entails externalities and may improve welfare. The other focuses on inefficiency caused by municipal mergers themselves. Various articles implies that municipal mergers may cause inefficiency because mergers will make room for municipalities to distort their decision. One example of such an inefficiency is the ’common pool problem’. Before mergers happen, municipalities to be merged issue more debt than optimal level since debt will be owed

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*1 Germany and South Africa, for example, experienced mandatory municipal mergers.
by other a merged municipality. This phenomenon is called ‘common pool problem’ and is empirically observed a lot. Regarding this point, municipal mergers will make inefficiency and may worsen social welfare, although the opposite effect is also expected by internalization of externalities. Then, do municipal mergers increase social welfare considering both the welfare improve effect by the internalization of externalities and the welfare worsen effect by the common pool problem? The first goal of this paper is to answer this question employing the model of tax competition.

The second aim of this paper is to investigate what kinds of policies are effective to improve welfare when mergers are implemented. Actually, as well as many papers about the common pool problem show, the result of this paper shows that municipal merger may not improve welfare since the effect from the common pool problem is large. This result implies that some policies to reduce the effect of the common pool problem should be considered to improve the welfare. Therefore, these policies are also examined in this paper.

Some articles about tax competition show that municipal mergers increase social welfare in the static model. This is mostly because municipal mergers terminate tax competition between municipalities. This conclusion has been well known from the beginning of the research about tax competition as Zodrow and Mieszkowski (1983) and Hoyt (1991) showed briefly. While these papers consider the reduction of the number of municipalities, municipal mergers also entail tax coordination, which is another solution for eliminating harmful tax competition. The fact that tax coordination also terminate tax competition is famous and many articles show this result\textsuperscript{2}. However, ordinal tax coordination among regions tends to fail because each region has an incentive to deviate from the coordination and has a chance to compete using other instruments\textsuperscript{3}. Tax coordination caused by mergers, on the other hand, does not have such a fear even when other policy variables than capital tax are considered\textsuperscript{4}. As a result, municipal mergers seem to be a very strong and realistic solution against tax competition.

However, municipal mergers may cause some problems in the dynamic context. In particular, the ‘common pool problem’, where municipalities issue more debts than the efficient level because debts can be owed by a merged municipality, has been pointed out. The common pool problem is often referred to practically and empirically. For example, Hinnerich (2009) and Jordahl and Liang (2010) show that Swedish municipalities had a tendency to issue more debts before they merged. Similar empirical studies show that the common pool problem happened in Denmark, Finland, Germany and Japan\textsuperscript{5}. In the theoretical context, there are also some studies about the common pool problem of local governments.

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\textsuperscript{2} Wilson (1986) and Konrad and Schjelderup (1999), for example, show this. Zodrow (2003) summarizes research about coordination among regions. In addition, Vrijburg and de Mooji (2016) examine mergers of higher governments in the two-tier government model. As explained later, these models do not consider dynamic context and do not entail public debt.

\textsuperscript{3} Fuest (1995), for example, introduces productive public services into tax competition model and shows that competition over public services can occur even if tax coordination is installed. Other papers like Fuest and Huber (2001) or Marchand et al. (2003) introduce other instruments and show partial tax or fiscal coordination do not always improve welfare.

\textsuperscript{4} Indeed, this paper introduce capital tax and debt as policy variables, and these variables are completely coordinated once municipal merger happens in the model.

\textsuperscript{5} Hansen (2014), Saarimaa and Tukiainen (2015), Fritz and Feld (2015), Nakazawa (2016), and Hirota and Yunoue (2017) are examples.
Weingast et al. (1981) is one of these researches. In their setup, a politician in area \( i \in N \) spends on a project for area \( i \) more than the optimal level because the cost of the project is shared by whole \( N \) areas. Tyrefors (2006) also shows the mechanism of the common pool problem of local governments by employing the two-period model of Persson and Tabellini (2002). These studies suggest that the common pool problem may distort the budget expenditures of municipalities.

If the effect from the common pool problem is strong enough, this bad effect will eclipse welfare improvement caused by the termination of tax competition. On the other hand, municipal merger may be a wonderful and realistic solution to cease harmful tax competition if the bad influence by the common pool problem is weak. Local tax competition is observed in reality\(^6\), so measures to suppress harmful tax competition are needed. Thus, it is important to examine whether municipal mergers improve welfare by policy coordination or worsen it by the common pool problem. However, capturing the effect caused by common pool problem is impossible in the existing static models. Therefore, I examine how welfare changes when a municipal merger happens in a dynamic tax competition model.

The literature about municipal merger and tax competition is rare in spite of its importance. The exception is Ogawa and Susa (2015). They investigate the model where municipal mergers are endogenously decided along with international tax competition, although they do not consider common pool problem or local tax competition\(^7\). Thus, there is no article which consider local tax competition and municipal merger as long as I know. This paper investigates the tax harmonization effect of municipal merger on local tax competition and common pool problem.

This paper analyzes whether the municipal merger increases social welfare employing the framework of Jensen and Toma (1991) and Matsumoto (2011), both of which analyze the two-period tax competition between two governments\(^8\). Jensen and Toma (1991) show that municipalities issue a debt (if tax is a strategic complement) because a debt-issuing action will reduce tax competition in the period two. Matsumoto (2011) introduces a vertical government structure where central and municipal governments have the same tax base. He shows that governments will generate not a debt but a surplus under this setting. These papers analyze tax competition in dynamic context while they do not consider municipal mergers. Thus, this paper considers the municipal merger using a very similar framework with these two papers. There are several differences between them and this paper. Especially in this model, the interest rate for debt and saving, \( r \), is exogenous, which makes the model much simpler, although this change does not affect the main result. In addition, tax rates are assumed as strategic complements in this model for simplicity since many articles about tax competition analyze the case of strategic complementary tax.

Comparing a welfare-increase effect of municipal merger (thanks to reduction of a tax competition) with a welfare-decrease effect of municipal mergers (owing to the existence of the common pool problem), this paper shows that the effect of the common pool problem might exceed the effect of reducing tax

\(^6\) Buettner (2003), for instance, observes tax competition among German local governments.  
\(^7\) They assume that tax rate of each jurisdiction is united, which makes the model have no tax competition between local governments.  
\(^8\) There are several models which deal with tax competition and debts or savings (Krelove, 1992; Keen and Kotsogiannis, 2002; Krogstrup, 2004; Batina, 2009) while the model installing endogenous debt issuance and public-goods-supply in each period is rare. An exception is Janeba and Todtenhaupt (2016) and they deal with two-period tax competition considering endogenous debt issuance while the their model setting is quite different from this paper.
competition in certain settings. The other main results of this paper are the following points: first, social welfare can be recovered employing the policies which reduce the effect from the common pool problem; second, governments issue excessive debts in both municipal merger and non-municipal merger cases; and third, over-issuing of the debts is caused by different reasons in each case. Specifically, anticipation of the other municipality produces over-issuing of the debts in a non-merger case and the discounting for marginal disutility of issuing debts by the common pool problem generates over-issuing in merger cases.

This paper consists of eight sections. The basic setting of this model is described in Section 2. The equilibrium conditions of public policy in the benchmark case, the non-municipal merger case, and the municipal merger case are investigated in Sections 3, 4, and 5, respectively. Section 6 focuses on the welfare comparison of each case using numerical calculation. Section 7 discusses the public policy to reduce social welfare loss. Section 8 presents concluding remarks.

2 Model

Consider an economy with two periods (denoted by $\tau = 1, 2$) and two identical municipalities (denoted by $i = 1, 2$). In each municipality, there is a government and a representative consumer who supply one unit of labor inelastically. A private good $x_{i\tau}$ and a public good $g_{i\tau}$ are consumed by the consumer in municipality $i$ of period $\tau$. The private good $x_{i\tau}$ is a numeraire, and can be consumed or used as an input for producing the public good $g_{i\tau}$. A consumer in $i$ at $\tau$ has the utility function $u_{i\tau} = u(x_{i\tau} + h(g_{i\tau}))$, where $u' > 0 > u''$ and $h' > 0 > h''$ are assumed. Since $x_{i\tau}$ is a numeraire and the marginal cost of $g_{i\tau}$ is one in this model, the condition for the efficient provision of goods is $h'(g_{i\tau}) = 1$.

Capital in $i$ at period $\tau$ is denoted as $k_{i\tau}$. Capital is completely mobile and is used as input of the production. In each period, capital endowment, $\bar{k}$, is distributed to each municipality and it can be invested in both municipalities while capital is fully depreciated, which means that capital cannot be used in the next period\textsuperscript{*9}. Production function per one unit of labor is $f(k_{i\tau})$, where $f' > f'' = 0 > f'''$ is assumed\textsuperscript{*10}.

The total capital stock in this economy in each period is $K$, which is exogenously given\textsuperscript{*11}. Thus, the equilibrium condition for the domestic capital market at $\tau$ is

$$k_{1\tau} + k_{2\tau} = 2\bar{k} = K. \tag{1}$$

\textsuperscript{*9} We can also interpret this setting as another setting where capital investors reallocate their capital in the beginning of each period. This is because residents, who are also investors, in both areas have the same amount of capital in each period and invest it to obtain its maximum return.

\textsuperscript{*10} The firm in municipality $i$ produces an output employing one unit of labor and $k_{i\tau}$ of capital. This firm solves the profit maximization problem $\max_{k_{i\tau}} \pi = f(k_{i\tau}) - w - pk$ and capital rent $p = f'(k_{i\tau})$ is derived from the first order condition of this problem. Since the economy is perfectly competitive, $\pi = 0$ can be derived. Thus, the wage level is determined as $w = f(k_{i\tau}) - f'(k_{i\tau})k_{i\tau}$. The government in municipality $i$ imposes capital taxation $t_{i\tau}$ on the capital employed in $i$. As a result, a resident receives a wage $f(k_{i\tau}) - f'(k_{i\tau})k_{i\tau}$ and the after-tax return of capital investment $k(f'(k_{i\tau}) - t_{i\tau})$.

\textsuperscript{*11} In this model, each area invites directly invested domestic capital, which originally owned by residents in both areas. This setting assumes that governments attract not international capital (or companies) but local capital (or companies). If we consider international capital here, the model will be complicated and off the point of this paper.
Each government raises revenue in each period by a unit tax \( t_{i\tau} \) on capital used in that municipality. On the other hand, because of capital arbitrage dealing, the after-tax return of capital investment is equalized as \( \rho_{\tau} \). Then,

\[
f'(k_{i\tau}) - t_{i\tau} = f'(k_{j\tau}) - t_{j\tau} = \rho_{\tau} \tag{2}
\]

holds as an arbitrage condition. Because \( \rho_{\tau} \) and \( k_{i\tau} \) are affected by \( t_{i\tau} \) and \( t_{j\tau} \), they can be written as

\[
k_{i\tau} = k_{i\tau}(t_{i\tau}, t_{j\tau}) \tag{3}
\]

Consumers buy the private good within their budget constraint. The net income of the consumer in \( i \) is

\[
y_{i\tau} = f(k_{i\tau}) - f'(k_{i\tau})k_{i\tau} + \rho_{\tau} \hat{k}
\]

where \( y_{i\tau} \) can be written as \( y_{i\tau} = y_{i\tau}(t_{i\tau}, t_{j\tau}) \). Consumers can save their income at period 1 and withdraw that saving at period 2. Thus, their budget constraint is

\[
\begin{align*}
x_{i1} &= y_{i1} - s_i \\
x_{i2} &= y_{i2} + (1 + r)s_i
\end{align*} \tag{4}
\]

where \( s_i \) is saving and \( r \) is interest rate for the saving.

As consumers can save their income, governments can issue the debt \( d_i \) at period 1 and it must be repayed at period 2. Then, the governmental budget is

\[
\begin{align*}
g_{i1} &= t_{i1}k_{i2} + d_i \\
g_{i2} &= t_{i2}k_{i2} - (1 + r)d_i
\end{align*} \tag{5}
\]

where \( r \) is the common interest rate for the debt as well as the saving. This setting implies the saving and the debt are dealt in the international market, then the interest rate is exogenous in this model\(^{12} \).

The total utility of consumer \( i \) is

\[
W_i \equiv u_{i1}(x_{i1} + h(g_{i1})) + \delta u_{i2}(x_{i2} + h(g_{i2})). \tag{6}
\]

Note that \( \delta \in [0, 1] \) is the discount factor for period 2. In order to maximize the utility of the consumer in each municipality, each government and consumer chooses \( \{t_{i1}, t_{i2}, d_i\} \) and \( s_i \), respectively, subject to the following time line (Figure 1).

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\(^{12}\) Though this setting may seem somewhat awkward considering the domestic capital market, this setting simplified the model, reflects some parts of the real world, and follows previous literature. Firstly, introducing endogenous interest rate \( r \) makes the model much more complicated and will not affect the result (See Jensen and Toma (1991) for endogenous setting.). Secondly, more and more local public debts are securitised and sold at the international bond market in many countries. For example, the local public debts of U.S. and Canada are originally and mainly dealt in the bond market. In addition, Japan and France introduced the market principle into their system of local public debt from 1990s (Doi, 2007). Thirdly, Jensen and Toma (1991) and Matsumoto (2011) assume that the market about capital \( k \) and the markets about saving \( s_i \) and \( d_i \) are different as well as this paper. (Jensen and Toma (1991) introduce endogenous \( r \), while the markets between \( k \) and the others (\( s_i \) and \( d_i \)) are still separated.) Then, this paper follows their setting. While Janeba and Todtenhaupt (2016) narrowly avoid this kind of setting, the mechanism used there is almost the same as employed in this paper.
The case where a municipal merger happens between period 1 and 2 is referred to as ‘municipal merger case’ (Section 5). The case where no merger happens is called a ‘non-municipal merger case’ (Section 4). In addition to them, the benchmark case where municipalities are merged in both periods is also examined (Section 3).

In each case, consumer $i$ decides the amount of saving solving the following utility maximization problem:

$$\max_{s_i} W_i = u_{i1}(y_{i1} - s_i + h(g_{i1})) + \delta u_{i2}(g_{i2} + (1 + r)s_i + h(g_{i2})).$$  \quad (7)

The first order condition of this problem is\(^{13}\)

$$u_i'(x_{i1} + h(g_{i1})) = (1 + r)u_i'(x_{i2} + h(g_{i2}))(1 + \frac{\partial y_{i2}}{\partial t_{i2}} \frac{\partial t_{i2}}{\partial s_i} + h'(g_{i2})\frac{\partial g_{i2}}{\partial t_{i2}} \frac{\partial t_{i2}}{\partial s_i}).$$  \quad (8)

Thus, the saving is decided as (8) is satisfied.

3 Benchmark case: Merged municipality throughout both periods

As a benchmark case, a merged municipality throughout both periods is examined. In this case, no tax competition happens because the same capital tax rate is applied in both areas throughout both periods.

The government decides $\{t_1, t_2, d\}$ in order to maximize the welfare of both areas. $t_\tau(\equiv t_{i\tau} = t_{j\tau})$ is the common tax rate for both areas and is chosen at period $\tau$. $d(\equiv d_i = d_j)$ is the common debt level for both areas and is decided at period 1.

In this first-best case, $\partial k_{i\tau}/\partial t_{\tau} = 0$ holds since both areas are identical and a differentiation of (1) derives

$$\frac{\partial k_{i\tau}}{\partial t_{\tau}} + \frac{\partial k_{j\tau}}{\partial t_{\tau}} = 0.$$

Then, from (2), we can derive

$$\frac{\partial \rho_{\tau}}{\partial t_{\tau}} = f''(k_{i\tau}) \frac{\partial k_{i\tau}}{\partial t_{\tau}} - 1 = -1.$$

\(^{13}\) We can check that $\frac{\partial G_{i\tau}}{\partial t_{\tau}} = 0$ holds later in each case of this paper. The reason why this holds is that $t_{i\tau}$ turns out to be the function which is irrelevant to $s_i$. 

6
Since the subgame perfect equilibrium is derived by the backward induction, the game is solved backwards. The utility maximization problem for the government at period 2 is

$$\max_{t_2} \sum_{i=1}^{2} u_i(x_{i2} + h(g_{i2})).$$  \(9\)

The result of the first order condition shows

$$h'(g_{i2}) = 1.$$  \(10\)

From this equation, the equilibrium level of a public good at period 2, \(g_{i2}^*\) is derived. Given this \(g_{i2}^*\) and (5), the equilibrium tax rate at period 2, \(t_2^*\), is derived as

$$t_2^* = g_{i2}^* + \frac{(1 + r)d}{k}.$$  \(11\)

We can see that \(t_2^*\) depends on \(d\), \(g_{i2}^*\), and exogenous parameters. Thus, we can derive the indirect utility function of period 2 like

$$V_{i2}(d) = \max_{t_2} u_{i2}(x_{i2} + h(g_{i2}^*)).$$

In addition to this, \(\frac{\partial t_2}{\partial s_1} = 0\) also holds from (11). Then, (8) is simplified as

$$u'_{i1}(x_{i1} + h(g_{i1})) = \delta (1 + r)u'_{i2}(x_{i2} + h(g_{i2})).$$  \(8'\)

Next, the problem at period 1 is examined. The utility maximization problem for the government at period 1 is

$$\max_{t_1,d} \sum_{i=1}^{2} W_i = \sum_{i=1}^{2} u_{i1}(x_{i1} + h(g_{i1})) + \delta V_{i2}(d).$$  \(12\)

From the first order conditions for this problem, we can get the results below

$$h'(g_{i1}) = 1$$  \(13\)

$$u'_{i1}h'(g_{i1}) - \delta u'_{i2}h'(g_{i2})(1 + r) = 0.$$  \(14\)

As a result, the condition for the efficient provision of goods, \(h'(g_{i1}) = h'(g_{i2}) = 1\), holds and \(d\) is decided as (14) is satisfied in the benchmark case. The first (second) term of (14) is considered as the marginal (dis)utility of debt issuance and this equation is the equivalent to the solution of the ordinal inter-temporal resource allocation problem.

4 Non-municipal merger case

I begin this section with an analysis about the effect of capital taxation in the non-municipal merger case. From (2), we can derive

$$f''(k_{i\tau}) \frac{\partial k_{i\tau}}{\partial t_{i\tau}} - 1 = f''(k_{j\tau}) \frac{\partial k_{j\tau}}{\partial t_{i\tau}}.$$

\(^{14}\) See appendix 9.1.

\(^{15}\) See appendix 9.2.
A differentiation about (1) shows
\[ \frac{\partial k_{i\tau}}{\partial t_{i\tau}} + \frac{\partial k_{j\tau}}{\partial t_{i\tau}} = 0. \]
Then,
\[ \frac{\partial k_{i\tau}}{\partial t_{i\tau}} = \frac{1}{f''(k_{i\tau}) + f''(k_{j\tau})} = \frac{1}{2f''(k_{j\tau})} \]
\[ \frac{\partial k_{j\tau}}{\partial t_{i\tau}} = \frac{f''(k_{i\tau}) + f''(k_{j\tau})}{2f''(k_{j\tau})} \]
\[ \frac{\partial p}{\partial t_{i\tau}} = -\frac{1}{2} \]
can be derived\(^{16}\).

The subgame perfect equilibrium is derived by the backward induction. The utility maximization problem for the government at period 2 is
\[
\max_{t_{i2}} u_{i2}(x_{i2} + h(g_{i2})). \tag{15}
\]
With defining new function \(z_i\), the first order condition shows that
\[
z_i(t_{i2}, t_{j2}, d_i) \equiv -\frac{1}{2} k_{i2} - \frac{1}{2} \bar{k} + h'(g_{i2})[k_{i2} + \frac{t_{i2}}{2f''(k_{i2})}] = 0 \tag{16}
\]
holds\(^{17}\). \(z_i(t_{i2}, t_{j2}, d_i)\) satisfies the second order condition, which means \(\partial z_i/\partial t_{i2} < 0\).

From (5) and \(k_{i\tau} = k_{i\tau}(t_{1\tau}, t_{2\tau})\), \(t_{i2}\) can be written implicitly as
\[
t_{i2} = t_{i2}'(k_{i2}(t_{i2}; t_{j2}), g_{i2}(d_i, k_{i2}(t_{i2}; t_{j2})))
\]
\[= t_{i2}^*(t_{j2}, d_i). \]
Since this can be applied to \(t_{j2}^*\),
\[
\begin{cases}
t_{i2}^* = t_{i2}^*(t_{j2}, d_i) \\
t_{j2}^* = t_{j2}^*(t_{i2}, d_j)
\end{cases}
\]
hold. Solving these equation,
\[(t_{i2}^*, t_{j2}^*) = (t_{i2}^*(d_i, d_j), t_{j2}^*(d_i, d_j)) \tag{17}
\]
can be derived in the equilibrium. Then, we can derive indirect utility as
\[V_{i2}(d_i, d_j) = \max_{t_{i2}} u_{i2}(x_{i2} + h(g_{i2}^*)). \]
Since we consider the symmetric equilibrium, we can focus on only \(t_{i2}^*\). Note that \(t_{i2}^*\) does not depend on \(s_i\). Then, \(\partial t_{i2}/\partial s_i = 0\) and (8') hold. In the symmetric equilibrium, \(\bar{k} = k_{i\tau}\) holds and (16) is reduced as
\[h'(g_{2}) = \frac{\bar{k}}{\bar{k} + \frac{\bar{k}}{2f''(k)}}. \tag{18}
\]
Since this paper only considers the case where tax rates are strategic complements, we make the following assumption regarding \(t_{i\tau}\).
\(^{16}\)Note that \(f''(k_{i\tau}) = f''(k_{j\tau})\) because \(f''' = 0\) holds, then the value of \(f''(k)\) is constant.
\(^{17}\)See appendix 9.3.
Assumption 1  Tax rates are strategic complements. This means that \( t_i \) is assumed to satisfy

\[
\frac{\partial^2 u_{i}}{\partial t_i \partial t_j} > 0.
\]

This assumption is quite natural since many articles about tax competition introduce or assume that tax rates are strategic complements\(^{18}\). Especially, empirical literature about local capital taxation usually confirm whether tax rates have a positive correlation (in other words strategic complements) in order to show that there is tax competition\(^{19}\).

This assumption can be applied to \( V_{i2} \) and

\[
\frac{\partial^2 V_{i2}}{\partial t_{i2} \partial t_{j2}} = \frac{\partial u'_{i2}}{\partial t_{j2}} \times z_i + u'_{i2} \frac{\partial z_i}{\partial t_{j2}} > 0
\]

holds. Because of \( z_i = 0 \) and \( u'_{i2} \geq 0 \), \( \frac{\partial z_i}{\partial t_{j2}} > 0 \) is derived from the equation above.

Given the indirect utility function of period 2, the utility maximization problem for the government at period 1 is

\[
\max_{t_{i1}, d_i} W_i = u_{i1}(x_{i1} + h(g_{i1})) + \delta V_{i2}(d_i, d_j).
\]

From the first order conditions of this problem,

\[
h'(g_{i1}) = \frac{k}{k + \frac{t_{i1}}{2f''(k)}} \quad (21)
\]

\[
u'_{i1} h'(g_{i1}) - \delta u'_{i2} (1 + r) h'(g_{i2}) - \delta u'_{i2} h'(g_{i2}) \frac{t_{i2}}{2f''(k_{i2})} \frac{\partial t^*_{j2}}{\partial d_i} = 0
\]

(22) can be derived\(^{20}\). From (8'), (22) is reduced as

\[
h'(g_{i1}) = h'(g_{i2})[1 + \frac{1}{1 + \frac{t_{i2}}{2f''(k_{i2})}} \frac{\partial t^*_{j2}}{\partial d_i}].
\]

(23) The equation (23) shows that \( h'(g_{i1}) < h'(g_{i2}) \) when \( \partial t^*_{j2}/\partial d_i > 0 \) holds and \( h'(g_{i1}) > h'(g_{i2}) \) when \( \partial t^*_{j2}/\partial d_i < 0 \) holds. As appendix 9.7 shows, \( \partial t^*_{j2}/\partial d_i > 0 \) can be derived when tax rates are strategic complements. Thus, the proposition below holds.

Proposition 1  In the symmetric subgame perfect equilibrium in the non-municipal merger case, each municipality issues a debt when the tax rate is the strategic complement.

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\(^{18}\) Vrijburg and de Mooji (2016) insist that tax rates can be strategic substitutes easily in the ordinal tax competition settings (They use CES utility and quadratic production.) and derive the condition where tax rates become strategic complements. While this paper use the different utility function in the numerical calculation, the range where tax rates are strategic complements is derived there tracing almost the same process as Vrijburg and de Mooji (2016) did.

\(^{19}\) Off course, yardstick competition can also explain this empirical phenomenon, though it is another problem from the assumption of strategic complement over tax competition.

\(^{20}\) See appendix 9.4.
(Proof) Since the tax rate is the strategic complement, \( \partial t_{i2}^* / \partial d_i > 0 \) holds. Given this, \( h'(g_{i1}) < h'(g_{i2}) \) can be derived from (23). Then, (18) and (21) show \( (1 + t_{i1} / 2f''(\bar{k})\bar{k})^{-1} < (1 + t_{i2} / 2f''(\bar{k})\bar{k})^{-1} \). Thus, \( t_{i2} > t_{i1} \) holds as a result.

On the other hand, \( g_{i1} > g_{i2} \) can be derived from \( h'' < 0 \). Considering this and the budget constraint of the government, \( d_i > 0 \) can be derived, which means each government issues positive debt. (Q.E.D.)

This proposition is equivalent to proposition 4 of Jensen and Toma (1991). The reason why this proposition holds is below.

Recall that (22) is

\[
    u_i'(g_{i1})h'(g_{i1}) - \delta u_{i2}'(1 + r)h'(g_{i2}) - \delta u_{i2}'h'(g_{i2}) \frac{t_{i2}}{2f''(g_{i2})} \frac{\partial t_{i2}^*}{\partial d_i} = 0. \tag{22}
\]

The first and third terms of this equation show the marginal utility of the debt issuance and the second term shows the marginal disutility of the debt issuance. Comparing the result from the benchmark case, the existence of the third term may increase the marginal utility of a debt, and this is perhaps the reason for \( d_i > 0 \). The third term shows the effect of \( i \)'s debt issuance on \( j \)'s tax rate at period 2. In this model, generating a debt is equivalent to the commitment to raise the tax rate at period 2 since the government has to pay it back later. Considering this commitment, the other government can increase the tax rate when the tax rate is a strategic complement and this tax increase will lead to reducing harmful tax competition. To summarize, making a commitment through debt issuance reduces harmful tax competition and is evaluated as excess marginal utility. Thus, the governments issue a debt in this case.

5 The municipal merger case

In this setting, both municipalities get merged at period 2 and they unify their tax rates as

\[ t_2 \equiv t_{i2} = t_{j2}. \tag{24} \]

The budget of the merged government is

\[ I \equiv t_2(k_{i2} + k_{j2}) - (1 + r)(d_i + d_j) \tag{25} \]

and \( i \)'s public goods are distributed as \( g_{i2} = I/2 \).

Before solving the model, this section starts from the analysis of relations among \( \rho, k, \) and \( t \). A differentiation of (1) derives

\[
    \frac{\partial k_{i2}}{\partial t_2} + \frac{\partial k_{j2}}{\partial t_2} = 0.
\]

This equation can be reduced as \( \partial k_{i2} / \partial t_2 = 0 \) since each area is symmetric. Then, considering a differentiation about (2) we can derive

\[
    \frac{\partial \rho}{\partial t_2} = f''(k_{i2}) \frac{\partial k_{i2}}{\partial t_2} - 1
    = -1.
\]
Note that the differentiation of $k$ or $\rho$ by the tax rate of period 1, $t_{i1}$, is the same as one in the non-municipal merger case.

The subgame perfect equilibrium is derived by the backward induction. The utility maximization problem for the government at period 2 is

$$\max_{t_2} \sum_{i=1}^{2} u_{i2}(x_{i2} + h(\frac{I}{2})).$$  \hspace{1cm} (26)

Because of the union tax rate, capital flight does not happen and $\bar{k} = k_{i2} = k_{j2}$ holds. Then,

$$h'(\frac{I}{2}) = 1$$  \hspace{1cm} (27)

can be derived from the first order condition of (26). Given that $I^*$ satisfies (27),

$$t_2^* = \frac{I^* + (1 + r)(d_i + d_j)}{2k}$$  \hspace{1cm} (28)

holds from (5) and (25). Then, denote the second period tax rate satisfying the equation above as $t_2^* = t_2^*(d_i, d_j)$. From this equation, $\partial t_2^*/\partial s_i = 0$ holds because $t_2^*$ is decided to be irrelevant to $s_i$. Therefore, $s_i$ satisfies (8'). In addition, indirect utility is derived as

$$V_{i2}(d_i, d_j) = \max_{t_2} \sum_{i=1}^{2} u_{i2}(x_{i2} + h(\frac{I}{2})).$$  \hspace{1cm} (29)

Given the indirect utility, the utility maximization problem for the government at period 1 is

$$\max_{t_{i1}, d_i} W_i = u_{i1}(x_{i1} + h(g_{i1})) + \delta V_{i2}(d_i, d_j).$$  \hspace{1cm} (30)

From the first order conditions for this problem,

$$h'(g_{i1}) = \frac{\bar{k}}{k + \frac{1}{2} f''(\bar{k})}.$$  \hspace{1cm} (31)

$$u'_{i1} h'(g_{i1}) - \frac{1}{2} h'(\frac{I}{2}) = 0$$  \hspace{1cm} (32)

can be derived. Using (8'), (32) will be transformed as

$$h'(g_{i1}) = \frac{1}{2} h'(\frac{I}{2}) = 0.$$  \hspace{1cm} (33)

From this and (27), $h'(g_{i1}) = 1/2$ holds. Substituting this value to (31),

$$t_{i1} = 2f''(\bar{k})\bar{k} < 0.$$  \hspace{1cm} (34)

Given these results, the proposition below holds.

**Proposition 2** In the symmetric subgame perfect equilibrium in the municipal merger case, each municipality sets its tax rate at period 1 as negative value and issues a debt.

*21 See appendix 9.5.
*22 See appendix 9.6.
(Proof) As it has been already shown, substituting \( h'(g_{i1}) = 1/2 \) to (31), \( t_{i1} = 2f''(\bar{k})\bar{k} < 0 \) must be satisfied. Now considering \( g_{i1} = t_{i1}k_{i1} + d_i > 0 \), where \( t_{i1} < 0 \) and \( k_{i1} > 0 \) hold, \( d_i \) must be positive. (Q.E.D.)

An explanation of this proposition is the following. Consider (32)*23:

\[
\begin{align*}
    & u'_i h'(g_{i1}) - \delta u'_{i2} \frac{1+r}{2} h'(g_{i2}) \\
    &= u'_i h'(g_{i1}) - \delta u'_{i2}(1+r)h'(g_{i2}) + \delta u'_{i2}h'(g_{i2}) \frac{1+r}{2} \\
    &= 0.
\end{align*}
\]

(32)

Look at the second equation. The first and the third terms denote marginal utility and the second term indicates marginal disutility for issuing a debt. Comparing this equation with (14), there is the third term which shows the extra marginal utility. This extra term is considered to be caused by the common pool of the debt in the merged municipality. Thus, the common pool problem still exists even if the municipal merger extinguishes harmful tax competition. In addition, both (32) and (22) show similar third terms. The difference between (32) and (22) is the existence of \((t_{i2}/(1+r)f''(k))(\partial t_{i2}/\partial d_i)\) and the amount of \(d_i\) may depend on this*24.

6 Welfare comparison

6.1 Specifications of functions

Before the welfare comparison, this section starts from the specifications of function \( u(.) \), \( h(.) \), and \( f(.) \) as below*25.

\[
\begin{align*}
    & u(x + h(g)) = \log(x + h(g)) \\
    & h(g) = g^{\frac{1}{2}} \\
    & f(k) = ak - \frac{b}{2}k^2
\end{align*}
\]

(35)

Using these specifications, the social welfare of the benchmark case, the municipal merger case, and the non-municipal merger case are compared in the following parts of this section. Throughout this section, \( r = 0, \delta = 1 \) is assumed for simplification purposes.

Since the total utility in area \( i \) is given as (6), substitutions of (35) into this realize

\[
W_i = \log(f(k_{i1}) + (k - k_{i1})f'(k_{i1}) - t_{i1}\bar{k} - s_i + g_{i1}^{\frac{1}{2}}) \\
    + \log(f(k_{i2}) + (k - k_{i2})f'(k_{i2}) - t_{i2}\bar{k} + s_i + g_{i2}^{\frac{1}{2}})
\]

(36)

where \( f(k) \) is the common production function for both periods. Though the social welfare is \( W_i + W_2 \), it can also be evaluated just as \( W_i \) since each municipality is symmetric. Thus, \( W_i \) is used as the criterion of social welfare in the following sections.

*23 Note that \( g_{i2} = \frac{1}{2} \) here.

*24 Note that (22') is \( u'_1 h'(g_{i1}) - \delta u'_{i2}(1+r)h'(g_{i2}) - \delta u'_{i2}h'(g_{i2}) \frac{1+r}{2} \times \frac{1}{(1+r)h'(c_{i2})} \partial u'_{i2} = 0.\)

*25 \( k \in [0, \frac{a}{b}] \) is assumed.
6.2 Benchmark case

As a benchmark case, the social welfare of the merged municipality throughout both periods is examined. Since \( h'(g_{1r}) = 1 \) is realized in this case, \( g_{1r} = 1/4 \) is derived from (35). Thus, the values of \( d_i \) and \( s_i \) can be derived. Substituting these values to the total utility,

\[
W_i = 2 \log(f(\bar{k}) + \frac{1}{4})
\]

(37)

is realized in this case.

6.3 Municipal merger case

In this case, (27) and (33) are satisfied. Then, using (35),

\[
\begin{align*}
  h'(g_{11}) &= \frac{1}{2} \iff g_{11} = 1 \\
  h'(g_{12}) &= 1 \iff g_{12} = \frac{1}{4}
\end{align*}
\]

(38)

are derived. Then, the values of \( d_i \) and \( s_i \) are determined. Substituting these values to the total utility,

\[
W_i = 2 \log(f(\bar{k}) + \frac{1}{8})
\]

(39)

is realized in this case.

6.4 Non-municipal merger case

In this case, (18), (21), and (23) are satisfied. Thus,

\[
\begin{align*}
  h'(g_{11}) &= \frac{k}{k + \frac{1}{2}f'(k)} \\
  h'(g_{12}) &= \frac{k}{k + \frac{1}{2}f'(k)} \\
  h'(g_{11}) &= h'(g_{12})[1 + \frac{1}{2f'(k)} \frac{\partial t_{12}}{\partial d_i}]
\end{align*}
\]

(40)

hold. The debt level and tax rates are dependent on \( \partial t_{12}/\partial d_i \)*26 and numerical calculations are employed to derive each endogenous parameter.

Remember that the assumption 1 that the tax rates are strategic complements requires the condition of \( \partial z_i/\partial t_{12} > 0 \). To satisfy this condition, the second period tax rate in the non-municipal merger case must be in a certain level"27. When \( b \geq 0.8 \), \( t_{12} \) does not satisfy this condition under the setting of \( \bar{k} = 1 \). Thus, we make the assumption below.

Assumption 2 The range of \( b \) is limited as \( b \in (0, 0.8) \) to satisfy the assumption 1 that the tax rates are strategic complements.

---

*26 The concrete procedure to derive \( \partial t_{12}/\partial d_i \) are shown in appendix 9.8.
*27 See appendix 9.9 for it and its derivation.
As a result, we can derive the value of \{t_{i1}, t_{i2}, d_i\} by determining the parameters, \{k, b, r\}, and the social welfare is also derived by numerical analysis. The numerical calculations give the results shown in table 1. These results are derived under the setting of \(k = 1\) and \(r = 0\). The results in the municipal merger case are also cited in table 1 for comparison.

<table>
<thead>
<tr>
<th>Case</th>
<th>Non-municipal merger</th>
<th>Municipal merger</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>(t_{i1})</td>
<td>-0.0711</td>
<td>-0.6</td>
</tr>
<tr>
<td>(t_{i2})</td>
<td>0.4092</td>
<td>1.85</td>
</tr>
<tr>
<td>(d_i)</td>
<td>0.3840</td>
<td>1.6</td>
</tr>
<tr>
<td>Welfare((W_i))</td>
<td>2 \log(f(k) + 0.1900)</td>
<td>2 \log(f(k) + 0.125)</td>
</tr>
<tr>
<td>(b)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>(t_{i1})</td>
<td>-0.1982</td>
<td>-1</td>
</tr>
<tr>
<td>(t_{i2})</td>
<td>0.5076</td>
<td>2.25</td>
</tr>
<tr>
<td>(d_i)</td>
<td>0.5571</td>
<td>2</td>
</tr>
<tr>
<td>Welfare((W_i))</td>
<td>2 \log(f(k) + 0.2004)</td>
<td>2 \log(f(k) + 0.125)</td>
</tr>
<tr>
<td>(b)</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>(t_{i1})</td>
<td>-0.3335</td>
<td>-1.4</td>
</tr>
<tr>
<td>(t_{i2})</td>
<td>0.7675</td>
<td>2.65</td>
</tr>
<tr>
<td>(d_i)</td>
<td>0.7165</td>
<td>2.4</td>
</tr>
<tr>
<td>Welfare((W_i))</td>
<td>2 \log(f(k) + 0.2053)</td>
<td>2 \log(f(k) + 0.125)</td>
</tr>
</tbody>
</table>

Note that the range of \(b < 0.8\) is assumed and these results can be derived irrelevant to \(a\). This is the reason these three numerical examples are selected. Remember \(b\) is equal to the absolute value of \(f''(k)\), which shows the slope of the marginal production. This can be considered as the degree of the reaction against capital flight. Then, we can interpret that the larger \(b\) becomes, the larger subsidy governments distribute at period 1 issuing \(d_i\) because of the degree of the marginal effect of capital flight on the marginal production.

As a result, the following proposition can be held.

**Proposition 3** Under the assumption 2, there is a symmetric subgame perfect equilibrium of the non-municipality merger case which generates higher social welfare than the one of the municipal merger case.

(Proof) Under the specifications such as (35), the social welfare realized in the municipal merger case is \(W_i = 2 \log(f(k) + 1/8)\). On the other hand, as table 1 shows, the social welfare in the non-municipal merger case becomes higher than \(W_i = 2 \log(f(k) + 1/8)\) within the positive range of \(b\) where assumption 2 is satisfied. (Q.E.D.)

Despite the results in many static models of tax competition, this proposition shows that a municipal merger does not always increase social welfare under the dynamic tax competition model. This shows that a municipal merger utilized as a tool to terminate tax competition may lead to an unfavorable result for the social welfare. Thus, it is important to solve the common pool problem when a municipal merger

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*28 Because \(a\), the parameter of the production function, is not employed to calculate the result of table 1, we can set \(a(> 0)\) arbitrarily satisfying the result of table 1. Of course, we can set a \(k\) value other than 1, while results are omitted for reasons of space.

*29 Note that issuing a large amount of \(d_i\) can be interpreted as a commitment to increasing a tax rate in period 2, which makes an incentive for the other municipality to increase a tax rate in period 2. Then, the tax competition at period 2 is relaxed as a result.
is considered.

7 Discussion and Policy implication

In this section, some options for solving the common pool problem in the municipal merger case are discussed.

As proposition 3 suggests, the distortion caused by the common pool problem is large enough to eclipse welfare improvement by mergers. However, continuous harmful tax competition should be eliminated employing municipal mergers with some following policies. Considering the common pool problem, the restriction of issuing the pooled debt is a straightforward and intuitive solution for eliminating the common pool problem because it produces the same effect as a tax coordination. The analysis in appendix 9.10 also shows that efficient conditions are satisfied when the pooled debt is completely restricted in the municipal merger case. Thus, the following proposition can be derived.

Proposition 4  In the municipal merger case, social welfare can be maximized when the pooled debt is completely restricted.

(Proof) See appendix 9.10. (Q.E.D.)

At a glance, this solution is a ideal and useful policy considering municipal mergers. However, the complete installation of this policy seems as difficult as the installation of tax coordination. While some Japanese municipalities, for instance, were prohibited from issuing debt if they had had deficits large enough to exceed some criteria, many of them still did issue debt before the mergers because they had not had deficits so large that they had been banned from issuing debt. Although Nakazawa (2016) shows the restriction was partly valid because some municipalities which had large deficits refrained from debt issuance, his result also shows that this policy does not work completely.

Another option for reducing the common pool problem is perhaps the restriction of distributing subsidies for the capital. Recollect that municipalities allocate subsidies at period 1, $t_{11} < 0$, for attracting the capital in the merger case. When the tax (subsidy) rate is zero ($t_{11} = 0$) in the merger case, the externality of tax competition is banished and efficiency conditions are satisfied\(^{30}\). However, the possibility of realizing this policy should be considered as well as the restriction for pooled debt.

8 Conclusion

This paper showed that municipal mergers do not always lead to efficient conditions because of the common pool problem in the dynamic model despite the results of the static model. As some research like Hinnerich (2009) or Nakazawa (2016) show, the common pool problem is a serious problem. With

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\(^{30}\) Substitute $t_{11} = 0$ into (31) and see that $h'(g_{11}) = 1$ holds. Off course $h'(g_{12}) = 1$ also holds since the problem at period 2 is solved for given $t_{11}$ and $d_i$. Note that, under the restriction of the subsidy, the optimization problem at period 1 has the corner solution.
reference to the discussion above, this paper also showed that the social welfare in the municipal merger case might be lower than the social welfare in the non-municipal merger case under certain settings. Therefore, it is important to consider a policy to resolve the common pool problem when using mergers as a tool to eliminate harmful tax competition. This paper considered some policies and showed that restrictions on pooling a debt are a possible policy for solving the common pool problem, though the installation of it seems difficult.

As an extension of this paper, there are several possible directions. While the choice of merger is not considered in this paper, the option to merge can be endogenized, which may explain the behavior of the municipality better. Increasing the number of municipalities in the model is one of the other extensions. Under the setting where the number of municipalities is over two, we can consider the existence of a municipality which does not join the merger. Although Konrad and Schjelderup (1999) show a result which concludes that a coordination between municipalities increases social welfare in the static setting, other results can be derived in the dynamic model. In order to obtain further insights about solutions for harmful tax competition, more sophisticated research is needed.

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9 Appendix

9.1 The derivation of (10)

The first order condition of (9) is

\[
\frac{\partial \sum_{i=1}^{2} u_{i2}}{\partial t_2} = \sum_{i=1}^{2} u'_{i2} \times \left( \frac{\partial u_{i2}}{\partial t_2} + h'(g_{i2})(t_2 \frac{\partial k_{i2}}{\partial t_2} + k_{i2}) \right) \\
= \sum_{i=1}^{2} u'_{i2} \times (\bar{k} + h'(g_{i2})k_{i2}) = 0.
\]

Considering \( u'_{i2} \neq 0 \) and \( \bar{k} = k_{i2} \) holds since there is no capital flight in the benchmark setting, (10) holds as a result.
9.2 The derivation of (13) and (14)

The first order conditions of (12) are

\[
\begin{align*}
\sum \frac{\partial W_i}{\partial t_i} + \sum \frac{\partial W_i}{\partial x} \frac{\partial x}{\partial t_i} &= 0 \\
\sum \frac{\partial W_i}{\partial d} + \sum \frac{\partial W_i}{\partial x} \frac{\partial x}{\partial d} + \sum \frac{\partial W_i}{\partial t_i} \frac{\partial t_i}{\partial d} &= 0.
\end{align*}
\]

(41)

Note that \( \frac{\partial W_i}{\partial t_i} = \frac{\partial W_i}{\partial s} = 0 \) holds from the consumer’s problem for the saving and \( \frac{\partial W_i}{\partial t_2} = 0 \) holds from the first order condition of the second period’s problem. Then the first order condition can be translated as

\[
\sum \frac{\partial W_i}{\partial t_i} = \sum_{i=1}^2 u_i' t_i \left( \frac{\partial y_i}{\partial t_i} + h'(g_i)(t_i \frac{\partial k_i}{\partial t_i} + k_i) \right)
\]

\[
= \sum_{i=1}^2 u_{i1}' (-\bar{k} + h'(g_{i1})k_{i1}) = 0
\]

(42)

\[
\sum \frac{\partial W_i}{\partial d} = \sum_{i=1}^2 u_{i1}' h'(g_{i1}) - \delta \sum_{i=1}^2 u_{i2}' h'(g_{i2})(1 + r) = 0.
\]

(43)

Note that \( \bar{k} = k_{i1} \) holds since there is no capital flight in this setting. In addition, \( u_{i1}' \neq 0 \) holds in this model. Then, (13) holds. As for (43), we can also get (14) since each area is symmetric.

9.3 The derivation of (16)

In (15), \( s_i \) and \( d_i \) are given. In addition, each municipality is symmetric. Given these conditions, the first order condition of this problem is

\[
\frac{\partial u_{i2}}{\partial t_2} = u_{i2}' \left( \frac{\partial y_{i2}}{\partial t_2} + h'(g_{i2})\left[k_{i2} + t_2 \frac{\partial k_{i2}}{\partial t_2}\right] \right)
\]

\[
= u_{i2}' \left( -\frac{1}{2} k_{i2} - \frac{1}{2} \bar{k} + h'(g_{i2})k_{i2} + \frac{t_2}{2f''(k_{i2})} \right) = 0.
\]

(44)

Considering \( u_{i2}' \neq 0 \), (16) is derived.

9.4 The derivation of (21) and (22)

The first order conditions for (20) about \( t_{i1} \) and \( d_i \) are the following equations respectively.

\[
\begin{align*}
\frac{\partial W_i}{\partial t_{i1}} + \frac{\partial W_i}{\partial x} \frac{\partial x}{\partial t_{i1}} &= 0 \\
\frac{\partial W_i}{\partial t_i} + \frac{\partial W_i}{\partial x} \frac{\partial x}{\partial t_i} + \frac{\partial W_i}{\partial t_{i2}} \frac{\partial t_{i2}}{\partial t_i} &= 0
\end{align*}
\]

(45)

\[^{31}\text{Note that } \frac{\partial W_i}{\partial t_{i2}} = \frac{\partial W_i}{\partial t_2} = 0 \text{ holds.}\]
From the optimization problem about savings, \( \partial W_i/\partial s_i = 0 \) holds. Thus the equations above can be reduced as

\[
\frac{\partial W_i}{\partial t_{11}} = u'_1 \times \left( \frac{\partial y_{i1}}{\partial t_{11}} + h'(g_{i1})[k_{i1} + t_{i1} \frac{\partial k_{i1}}{\partial t_{11}}] \right)
= u'_1 \times \left( -\frac{1}{2} k_{i1} - \frac{1}{2} \hat{k} + h'(g_{i1})[k_{i1} + \frac{t_{i1}}{2f''(k_{i1})}] \right) = 0
\]

(46)

\[
\frac{\partial W_i}{\partial d_i} + \frac{\partial W_i}{\partial t_{j2}} \frac{\partial t_{j2}}{\partial d_i} = u'_1 h'(g_{i1}) - \delta u'_{i2}(1 + r)h'(g_{i2}) + \delta \frac{\partial V_{i2}}{\partial t_{j2}} \frac{\partial t_{j2}}{\partial d_i} = 0.
\]

(47)

Considering \( u'_1 \neq 0 \) and \( k = k_{ir} \), (46) will be (21).

On the other hand,

\[
\frac{\partial u_{i2}}{\partial t_{j2}} = u'_2 \times \left( \frac{\partial y_{i2}}{\partial t_{j2}} + h'(g_{i2}) \frac{\partial k_{i2}}{\partial t_{j2}} \right)
= -u'_2 \times h'(g_{i2}) \frac{t_{i2}}{2f''(k_{i2})}
\]

can be derived in the symmetric equilibrium. Then, (22) holds from (47).

9.5 The derivation of (27).

Thus, the first order condition for (26) is

\[
\sum \frac{\partial u_{i2}}{\partial t_{j2}} = \sum u'_2 \times \left( \frac{\partial y_{i2}}{\partial t_{j2}} + \frac{h'(g_{i2})}{2} \frac{\partial I}{\partial t_{j2}} \right)
= \sum u'_2 \times \left[ -\hat{k} + \frac{h'(g_{i2})}{2} (k_{i2} + k_{j2}) \right] = \sum u'_2 \hat{k}[-1 + h'(\frac{I}{2})] = 0.
\]

(48)

From \( u'_{i2} \neq 0 \), the equation above can be reduced as (27).

9.6 The derivation of (31) and (32)

The first order conditions of (30) are

\[
\left\{
\begin{array}{l}
\frac{\partial W_i}{\partial t_{i1}} + \frac{\partial W_i}{\partial s} \frac{\partial s}{\partial t_{i1}} = 0 \\
\frac{\partial W_i}{\partial d_i} + \frac{\partial W_i}{\partial s} \frac{\partial s}{\partial d_i} + \frac{\partial W_i}{\partial \sigma} \frac{\partial \sigma}{\partial d_i} = 0.
\end{array}
\right.
\]

Remember that by optimizing the problem about saving, \( \partial W_i/\partial s_i = 0 \) can be derived and (48) shows that \( \partial W_i/\partial t_{j2} = 0 \) holds. The first order conditions can be reduced as

\[
\frac{\partial W_i}{\partial t_{i1}} = u'_1 \times \left( \frac{\partial y_{i1}}{\partial t_{i1}} + h'(g_{i1})[k_{i1} + t_{i1} \frac{\partial k_{i1}}{\partial t_{i1}}] \right)
= u'_1 \times \left( -\frac{1}{2} k_{i1} - \frac{1}{2} \hat{k} + h'(g_{i1})[k_{i1} + \frac{t_{i1}}{2f''(k_{i1})}] \right) = 0
\]

(49)

\[
\frac{\partial W_i}{\partial d_i} = u'_1 h'(g_{i1}) - \delta u'_{i2}(1 + r)h'(\frac{I}{2}) = 0.
\]

(32)

\[\text{Note that } \frac{\partial W_i}{\partial \sigma} = \frac{\partial W_i}{\partial s} = 0.\]
As for (49), applying $u'_{i1} \neq 0$,

$$- \frac{1}{2} k_{i1} - \frac{1}{2} k + h'(g_{i1})[k_{i1} + \frac{t_{i1}}{2f''(k_{i1})}] = 0$$

$$h'(g_{i1}) = \frac{\frac{1}{2} k_{i1} + \frac{1}{2} k}{k_{i1} + \frac{t_{i1}}{2f''(k_{i1})}}$$  \hspace{1cm} (50)

can be derived. In the symmetric equilibrium, we get (31) since $k_{i1} = \tilde{k}$ holds.

9.7 The derivation of $\frac{\partial t_{j2}}{\partial d_i} > 0$ in non-merger case

From the total differentiation of (16), we can derive

$$\frac{\partial t_{j2}}{\partial d_i} = \frac{\partial z_i}{\partial d_i} \frac{\partial z_j}{\partial t_{j2}}$$  \hspace{1cm} (51)

where

$$\Phi = \frac{\partial z_i}{\partial t_{j2}} \frac{\partial z_j}{\partial t_{j2}} - \frac{\partial z_i}{\partial t_{j2}} \frac{\partial z_j}{\partial t_{j2}}$$  \hspace{1cm} (52)

Since the stability condition of the equilibrium at period 2 shows

$$| \frac{\partial z_i}{\partial t_{j2}} | > | \frac{\partial z_j}{\partial t_{j2}} |,$$

$\Phi > 0$ can be derived in the symmetric equilibrium. Therefore, in order to know the sign of $\frac{\partial t_{j2}}{\partial d_i}$, the signs about $\frac{\partial z_i}{\partial d_i}$ and $\frac{\partial z_j}{\partial t_{j2}}$ should be derived. The partial differentiation of $z_i$ by $d_i$ is

$$\frac{\partial z_i}{\partial d_i} = -h''(g_{i2})(1 + r)[k_{i2} + \frac{t_{i2}}{2f''(k_{i2})}] > 0.$$  \hspace{1cm} (53)

About $\frac{\partial z_j}{\partial t_{j2}}$, assumption 1 shows a similar equation to (19) and $\frac{\partial z_j}{\partial t_{j2}} > 0$ is satisfied. Then,

$$\frac{\partial t_{j2}}{\partial d_i} = \frac{\partial z_i}{\partial d_i} \frac{\partial z_j}{\partial t_{j2}} \frac{\partial t_{j2}}{\partial d_i} > 0$$  \hspace{1cm} (54)

can be derived.

9.8 The derivation of $\frac{\partial t_{j2}}{\partial d_i}$

In order to determine the value of $\{t_{i1}, t_{i2}, d_i\}$, the value of $\frac{\partial t_{j2}}{\partial d_i}$ comes to be important. So this value is investigated below. Since $\frac{\partial t_{j2}}{\partial d_i}$ is

$$\frac{\partial t_{j2}}{\partial d_i} = \frac{\partial z_i}{\partial d_i} \frac{\partial z_j}{\partial t_{j2}} \frac{\partial t_{j2}}{\partial d_i}$$,  \hspace{1cm} (51)
I examine the partial differentiation about $z_i$. Then,
\[
\frac{\partial z_i}{\partial t_{i2}} = -\frac{1}{2} \frac{\partial k_i}{\partial t_{i2}} + h''(g_{i2})(k_{i2}) + \frac{t_{i2}}{2f''(k_{i2})} + h'(g_{i2}) \left( \frac{\partial k_i}{\partial t_{i2}} + \frac{1}{f''(k_{i2})} \right)
\]
\[
\frac{\partial z_i}{\partial t_{j2}} = -\frac{1}{2} \frac{\partial k_i}{\partial t_{j2}} + h''(g_{i2})(k_{i2}) + \frac{t_{i2}}{2f''(k_{i2})} \frac{\partial k_i}{\partial t_{j2}} + h'(g_{i2}) \frac{\partial k_i}{\partial t_{j2}}
\]
\[
\frac{\partial z_i}{\partial d_i} = -h''(g_{i2})(1 + r)(k_{i2}) + \frac{t_{i2}}{2f''(k_{i2})}
\]
hold. Note that

\[
\begin{align*}
f'(k) &= a - bk \\
f''(k) &= -b \\
h'(g) &= \frac{1}{2} g^{-\frac{1}{2}} = \frac{1}{2} h(g)^{-1} \\
h''(g) &= -\frac{1}{4} g^{-\frac{3}{2}} = -2h'(g)
\end{align*}
\]
are realized from (35). In addition, $k = k_{ir}$ holds on the equilibrium**33. Given these, (18) and (21) are reduced as

\[
h'(g_{ir}) = \frac{\bar{k}}{k + \frac{t_{ir}}{2f''(k)}}
\]
\[
= \frac{\bar{k}}{k - \frac{t_{ir}}{2b}}.
\]
(55)

Using this, $\partial z_i/\partial t_{i2}, \partial z_i/\partial t_{j2}$, and $\partial z_i/\partial d_i$ become

\[
\frac{\partial z_i}{\partial t_{i2}} = \frac{1}{4b} - 2\bar{k}^2 \left( \frac{\bar{k}}{k - \frac{t_{ir}}{2b}} \right) - \frac{1}{b} \left( \frac{\bar{k}}{k - \frac{t_{ir}}{2b}} \right)^2
\]
\[
\frac{\partial z_i}{\partial t_{j2}} = \frac{1}{2b} \left( -\frac{1}{2} - 2t_{i2}k \left( \frac{\bar{k}}{k - \frac{t_{ir}}{2b}} \right)^2 + \left( \frac{\bar{k}}{k - \frac{t_{ir}}{2b}} \right) \right)
\]
\[
\frac{\partial z_i}{\partial d_i} = 2\left( \frac{\bar{k}}{k - \frac{t_{ir}}{2b}} \right)^2 (1 + r)\bar{k}
\]

Since each municipality is symmetric, $\partial z_i/\partial t_{i2} = \partial z_j/\partial t_{j2}, \partial z_i/\partial t_{j2} = \partial z_j/\partial t_{i2}$ holds and the value of $\partial t_{j2}^*/\partial d_i$ can be derived using the equations above.

**33** Because $k$ is not differentiated in the following part of this paper, there is no problem in substituting $\bar{k}$ as $k = k_{ir}$. 

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9.9 The derivation of the condition for $\frac{\partial z_i}{\partial t_{j2}} > 0$

The sufficient and necessary condition of $\frac{\partial z_i}{\partial t_{j2}} > 0$ is the following.

$$\frac{\partial z_i}{\partial t_{j2}} = \frac{1}{2b} \left( -\frac{1}{2} - 2\ell_2 k \left( \frac{k}{k - 2b} \right)^2 + \frac{k}{k - 2b} \right) > 0$$

$$\Leftrightarrow \frac{k}{k - 2b} > 2\ell_2 k \left( \frac{k}{k - 2b} \right)^2 + \frac{1}{2}$$

$$\Leftrightarrow k \left( k - \frac{\ell_2}{2b} \right) > 2\ell_2 k^3 + \frac{1}{2} \left( k - \frac{\ell_2}{2b} \right)^2$$

$$\Leftrightarrow \frac{1}{2} k^2 - 2\ell_2 k^3 - \frac{\ell_2^2}{8b^2} > 0$$

$$\Leftrightarrow \ell_2^2 + 16b^2 k^3 \ell_2 - 4b^2 k^2 < 0$$

$$\Leftrightarrow -8b^2 k^3 - 2bk \sqrt{16b^2 k^4 + 1} < \ell_2 < -8b^2 k^3 + 2b k \sqrt{16b^2 k^4 + 1} \quad (56)$$

Then, $\ell_2 \in (0, -8b^2 k^3 + 2bk \sqrt{16b^2 k^4 + 1})$ must be satisfied.

9.10 The restriction of pooled debt

Consider that only $\lambda \in [0, 1]$ part of debts is pooled in the budget of the merged municipality in the municipal merger case. (In this setting, the rest of debts of $i$, $(1 - \lambda)d_i$ is repayed by the capital tax in area $i$.) Thus, the budget of the merged municipality becomes

$$I = t_2(k_{j2} + k_{j2}) - \lambda(d_i + d_j). \quad (57)$$

Then, the budget in area $i$ at period 2 is

$$g_{i2} = \frac{I}{2} - (1 - \lambda)d_i. \quad (58)$$

Note that the union tax rate is applied at period 2 because the municipalities are merged then. So, (24) is satisfied.

Since subgame perfect equilibrium is derived by the backward induction, the game is solved backwards.

The utility maximization problem for the government at period 2 is

$$\max_{t_2} \sum_{i2}^2 u_{i2}(x_{i2} + h(g_{i2})). \quad (59)$$

The first order condition for (59) is

$$\frac{\partial \sum_{i2}^2 u_{i2}}{\partial t_{i2}} = \sum_{i2}^2 u'_{i2} \times [\frac{-k + h'(g_{i2})}{2}(k_{i2} + k_{j2})]$$

$$= \sum_{i2}^2 u'_{i2} \times [\frac{-k + h'(g_{i2}) \times \bar{k}}] = 0. \quad (60)$$

Then,

$$h'(g_{i2}) = 1 \quad (61)$$
can be derived. Note that $\partial t_2/\partial s_i = 0$ holds and (8') is satisfied.

Given the result of the optimization problem at period 2, the utility maximization problem for the government at period 1 is

$$\max_{t_{1i}, d_i} W_i = u_{i1} (x_{i1} + h(g_{i1})) + \delta V_{i2}. \quad (62)$$

The first order conditions for (62) are

$$\frac{\partial W_i}{\partial t_{1i}} = u_{i1}' \times \left( \frac{\partial y_{i1}}{\partial t_{1i}} + h'(g_{i1})(k_{i1} + t_{1i} \frac{\partial k_{i1}}{\partial t_{1i}}) \right) = 0 \quad (63)$$

$$\frac{\partial W_i}{\partial d_i} = u_{i1}' h'(g_{i1}) - \delta (1 + r) u_{i2}' h'(g_{i2}) \times \left( \frac{1}{2} \lambda (1 - \lambda) \right) = 0. \quad (64)$$

About (63), since $\bar{k} = k_{i1}$ holds in the symmetric equilibrium, (65) can be derived. As for (64), we can apply (8'). Then,

$$h'(g_{i1}) = \frac{\bar{k}}{k + \frac{1}{2} h''(k)} \quad (65)$$

$$h'(g_{i1}) = h'(g_{i2}) \times (1 - \frac{1}{2} \lambda) \quad (66)$$

can be derived.

When $\lambda = 0$ holds, which means pooled debt is completely restricted, (66) is reduced as $h'(g_{i1}) = h'(g_{i2})$. Considering this and the fact that (61) shows $h'(g_{i2}) = 1$, $h'(g_{i1}) = 1$ can be derived.

In addition, $t_{1i} = 0$ can be derived from $h'(g_{i1}) = 1$ and (65). This means that each government does not levy a tax at period 1 and instead imposes a tax at period 2, where there is no anxiety regarding tax competition.

**References**


