Counter Collusive Effect of Leniency Program

Sangwon Park *

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Abstract

The leniency program is to reduce the sanction if a wrongdoer voluntarily confesses his behavior or cooperates extraordinarily with investigation authority. To study the effect of the program for self-report on collusion, we consider a repeated procurement auction model where each colluder simultaneously chooses ‘confess’ or ‘no confess’ after every bidding. Unlike the previous literature which emphasizes no or only negative effect, it shows that the leniency program can be counter-collusive. The simple intuition is that a deviator of the collusion will get less sanction through his/her self-report, which makes the deviation less costly and collusion sustainment more difficult.

1 Introduction

Collusion is an explicit or implicit agreement on their actions between multiple individuals, to limit open competition among them. Due to its intrinsic nature, economists have much interested in the incentive problem among the competitors but relatively little research has been made on public authority’s effort and its effectiveness against collusion. However, every country has used a lot of resource to detect and prevent collusion, because not only it is morally unfair but also it deters efficiency of market system.

The leniency program is one of the government’s method to deal with collusion. It is a promise to reduce the sanction if a wrongdoer voluntarily confesses his behavior or cooperates extraordinarily with investigation authority.

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In analyzing the program, two kinds of reduction should be distinguished each other. One is offered to a firm who spontaneously self-reports collusion even before the public authority has started any investigation. Voluntary confess is helpful especially when collusion are very hard to detect. The other is applied to a firm who cooperates when investigation is underway, for example by providing a hard evidence to the public authority. Because the law in many countries requires a strong evidence in proving a cartel and it may be one thing to know its existence and another to prove it at court, cooperation with investigation also worth generous treatment. For the sake of explanation, let us call the former ‘self-report’ and the latter ‘cooperation’.

The various leniency programs have been used in U.S., European union and other countries such as Korea. In 1978, the US Antitrust Division of the Department of Justice first introduced an Amnesty program and allowed for avoiding criminal sanctions if the firms reveal the existence of cartel before the opening of any investigation. In 1993, the US redesigned the program such that the leniency was granted automatically to the first firm who reports the illegal activity. In addition, even after an investigation is under way, a firm who decides to cooperate can be given reduction of the sanction. In 1996, the European Union introduced a similar program in which the different level of reduction in fine were granted to the firms depending on whether the cooperation was made before or after an inquiry of public authority. In 2001, the EU also tried to enhancing the program by improving the transparency and certainty of leniency conditions. In Korea,...

Several previous studies analyzed and explained the effect of the program on collusion. They observed that in reality the risk of detection is rather low but the probability of conviction, given detection, is high. Their results can be summarized as follows.

First, the reduction for ‘cooperation’ can be effective in decreasing time and cost of proving collusion. The colluders have the incentive to cooperate because the expected fine

\footnote{The brief introduction of the leniency program in U.S. and EU is based on Brisset and Thomas (2004) and Motta and Polo (2003).}
is high once investigation has been carried out. On the contrary, the program itself may have a pro-collusive effect as explained in Motto and Polo (2003) and Brisset and Thomas (2004). Ex ante, the firms see collusion less risky because a smaller fine is imposed even though it is detected. These two conflicting effects require that the structure of the program should be sophisticated.

Second, as Spagnolo (2000b, 2004) and Brisset and Thomas (2004) argue, the reduction for self-report has no or even a positive effect in promoting collusion. Because of slight probability of being detected ex ante, the expected fine before any investigation is small, which means the benefit of self-report is also weak. However, because the future collusive profit will be lost by any self-report, the colluders will have no incentive to utilize the program. Buccirossi and Spagnolo (2006) and Spagnola (2000b) also argue that the reduction of the fine in case of self-report may work in a pro-collusive way by allowing the colluders to have a credible threat to punish a collusion deviator. That is, the colluders’ strategy to report their bad behaviors in case of any deviation from collusion becomes credible when they can enjoy the reduction of the fine after self-report. Therefore introduction of leniency program can make a collusion be sustained which would not be possible before.


This paper analyzes the effect of leniency for self-report in repeated interactions and shows that it can be counter-collusive, in contrast to the previous research. The results depend on the two characteristics of the model, the dynamic consideration in collusion formation and the strategic behaviors in the colluders’s self-report decision.

Consider an infinitely repeated interaction among the potential colluders. After their collusive actions are made at each period, they have a chance to self-report them to the

\footnote{To make it effective, Spagnolo (2000a) proposes that a stronger incentive should be provided such as even rewarding a confessor positive amount of money.}

\footnote{Brisset and Thomas(2004) is similar to this paper by adapting a repeated auction model but they could not capture all the relevant strategic factors in analyzing self-report.}
public authority.

(( Table 1 )) Strategic Self-report

<table>
<thead>
<tr>
<th></th>
<th>With the program</th>
<th>Without the program</th>
</tr>
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<tbody>
<tr>
<td>After no deviation in collusion</td>
<td>No self-report (*)</td>
<td>No self-report (**)</td>
</tr>
<tr>
<td>After a deviation</td>
<td>Self-report by everybody (***)</td>
<td>No self-report (****)</td>
</tr>
</tbody>
</table>

If the collusion successfully has been maintained, no one will want to unilaterally self-report regardless of existence of the leniency program because it will just wipe out the chance of the future collusive profit. (See (*) and (**) in the above table.) However, any deviation in the previous interactions makes the story quite different. To sustain the collusion any deviational behavior must be punished and typical method is to play the competitive action ever since. Therefore, there will be no future collusive profit after any deviation and the self-report becomes the tempting option to the colluders under the leniency program. Specifically, under the program, self-report is the dominant strategy among the colluders after any deviation under some parameters value. (See (***)). In contrast, without the program, it is meaningless to confess, as long as detection rate by the public authority is lot high enough. (See (****).) The effect of leniency program on collusion sustainment occurs thorough these different outcomes after deviation, with and without the program.

The different outcomes under two regimes in terms of self-report after a deviation can affect the deviator’s payoff and change his incentive to deviate. If no self-report is made, the deviator’s expected cost from public sanction is the detection rate times the full fine. In contrast, if self-report is chosen by everybody, his cost from public sanction becomes the reduced fine. Depending the relative size of two values, the existence of the leniency program can be interpreted differently as to collusion formation.

Firstly, it may mean that a deviator himself will choose self-report to reduce the expected sanction. This strategy works if the reduced sanction after self-report is smaller than that
expected sanction without any report. In this case, the leniency program itself lets the deviation more profitable and plays a negative role in sustaining collusion. This finding is main contribution of this paper and has not pointed out by previous study, as far as we know.

Secondly, it may mean that collusion participant can curb deviation from collusion by self-reporting in case of deviation. This threaten is credible because strategic structure in reporting stage is that of Prisoner’s Dilemma and self-report is the dominant strategy. The leniency program can hurt the deviator if the reduced sanction with self-report is still larger than the expected sanction without it. Therefore existence of leniency program can play a positive role in sustaining collusion and be pro-collusive.

The remainder of the paper is organized as follows. Section 2 introduces the basic model which potential collusion members participate in a infinitely repeated procurement auction. In section 3 we present the effect of the program on the collusion formation which is sustained through Nash reversion strategy. Section 4 allows a collusion to have more generous punishment than Nash reversion and shows that the similar results hold.

2 Basic Model

We adapt a standard model of repeated procurement auction. There are one buyer and \(n\) sellers who interact infinitely many times. The buyer wants to consume one unit of goods at each period. Each seller’s cost of producing the goods at each period follows independent, identical and continuous distribution and its support is \([0, c]\). For the simplicity, we assume there is no reservation price or it is very large. Each seller wants to maximize the lift time profit and \(\delta\) is the discount factor. \(\delta\) is assumed to be bigger than \(\frac{1}{2(1-\mu)}\) where \(\mu \in (0, 1)\) is defined later.

There may exist multiple Nash equilibria in the given stage game. Multiplicity of static Nash equilibrium does not cause any problem in deriving any result of this model. However,
to simplify the argument and notations, we only consider the symmetric Nash equilibrium as the benchmark of non-collusive outcomes. Let $s^{\text{ns}}$ be the symmetric Nash equilibrium in the stage game auction, $b(s^{\text{ns}})$ be the bidding strategy profile under $s^{\text{ns}}$, and $\pi^{\text{ns}}$ be the stage game payoff to each bidder.

It is well known that the degree of collusion possibility depends on how much information on the previous bidding is available to the colluders. Less information implies that a secretive deviation from the agreed actions is possible without triggering any punishment by other members. Here, we assume that identity of the winner and the winning price of each period are public information but other bidders’ bidding are hidden.\footnote{This information structure is not only convenient but also realistic.}

Formally, the information available to every bidder just before presenting the bidding at $t$ period is denoted by $h^t = (h^{t-1}, w^{*t-1}, b^{*t-1}, \alpha^t)$, where $w^{*t-1}$ and $b^{*t-1}$ are the winner and the winning bid at period $t - 1$, respectively. $\alpha^t$ is realization of a random variable and it follows an independent and identical distribution every period. We introduce $\alpha^t$ to model the situation that the winner at each period in the collusion may rely on a public randomization device. For example, at each period, the agreed winner is determined according to the outcome of a coin toss or the sun spot location.

The standard approach on the tie-breaking rule of determining the winner among the highest bidders is to assume the equal probability. However if we allow the possibility of probabilistic winning, then given the opponent bid of $b$ a player’s best response may not exist because it should be close enough to but still lower than $b$. To avoid this technical non-existence problem, we assume that the order of tie-breaking among the buyers is predetermined.\footnote{Without loss of generality we can say the order is $1 \to 2 \to 3 \to \ldots \to n$. For example, if buyer 2, 4 and 5 are the highest bidder, then buy 2 becomes the winner.}

If, at $t$ period, the sellers’ bidding are agreed, then there is risk of detection by the public authority. Detection is not perfect in the sense that it succeeds probabilistically. Let
\( \mu \) be the probability of successful detection conditional on the existence of collusion and \( F \) be the fine in case of a successful detection. Because we are only interested in modeling a self-reporting behavior, cooperative behaviors during the legal process of verifying at court are ignored and the detection of the collusion immediately implies its proof.

There could be a sophisticated collusion such that the competitive low price and the agreed high price appear in a cycle along the equilibrium. In reality, even at the stage of a competitive low price, this may be the target of detection by the public authority because any agreed future non-competitive bidding is regarded as collusion. However, in this model, we assume detection and public punishment are conducted only when the non-competitive high bidding has been used at this period.

In addition, it is assumed that even when some colluders deviate from the agreed high price at the current period, collusion may be detected after the bidding. A deviation does not mean that the winning price is the competitive one and any non-competitive bidding should be the object of public investigation. For example, when the colluded strategy prescribes that the agreed winner would bid \( m \) and and others are supposed to bid a higher price than \( m \), the best deviation would be bid a little less than \( m \) which is still far larger than the competitive bidding.

Lastly, after the collusion is detected, the competitive bidding will prevail forever. Theoretically, the players may organize another collusion in the future, specially when the punishment is a small financial fee, not the permanent deprivation of market participation. However, because a market with a collusion history will be the subject of thorough monitoring by the public authority, it is very unrealistic to allow another possibility of collusion.

To understand its strategic structure, we incorporate the stage of choosing whether to self-report or not. The colluders have an opportunity to confess to the public authority after the announcement of the bidding results and the detection by the public authority.
It is assumed that every colluder simultaneously decides whether to confess. Let $\sigma_i = (b_{it}(h^t), r^i_t(h^t, w^{*t}, b^{*t}))$ be bidder $i$’s strategy at period $t$. $h^t$ is the information as it is explained before and $r^i_t(h^t, w^{*t}, b^{*t})$ is the confess strategy which is to choose ‘confess’ or ‘no confess’. When there is at least one confessor, the penalty $F$ is imposed to every collusion member except confessor(s) whose fine will be reduced to $\alpha(k)F$. Let $\alpha(k)$ be the number of confessors and $0 < \alpha(1) < ... < \alpha(n) < 1$ hold. The exemption amount depends on how many colluders have confessed at the same time.

To avoid the technical problem of existence of multiple equilibrium, we introduce small confession cost. That is, the colluders have to pay $\varepsilon$ for their own self-report. In particular, we assume throughout the paper $\varepsilon < (\mu - \alpha(1))F$ holds. It implies that the cost in case of an unilateral confess, $\varepsilon + \alpha(1)F$, is smaller than the expected cost resulting from the detection by the public authority, $\mu F$. The followings sum up the sequence of actions in a given period.

- $n$ bidders simultaneous show their bid.
- The winer and the winning bid are announced.
- In case of collusion, the colluders simultaneously choose ‘confess’ or ‘no confess’.
- If no confess is chosen by all colluders, the random detection by the public authority is realized.

We still need to specify the scope of collusion. It, in principle, can be any form of a non-competitive agreement but we need to impose structures to get meaningful results. Specifically, the collusion, denoted by $d$, is represented by an equilibrium of the repeated auction with the following properties (a)-(d). The equilibrium concept we adapt in this paper is perfect Baysian Nash Equilibrium.

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6It includes, for example, transportation fee to the public authority office.
• (a) There is one agreed winning bid, \( m \), at every period and and \( \sigma < m \).

• (b) Every player participates in the collusion. The winning bidder at each period is determined before presenting the bid and denoted by \( w^t(\alpha^t; d) \).

• (c) If, up to \( t - 1 \) period, the actual winners and the winning prices have been the same as the agreed ones and the collusion has not been detected by public authority, then the collusion is maintained at period \( t \).

• (d) Suppose that for the first time at \( t' - 1 \) the realized outcome is different from the colluded one. The collusion prescribes that the each bidder’s bidding is greater than or equal to the one under \( s^{ns} \). Formally, \( b^t_{i'}(c^t_{i'} + t | d) \geq b_i(c^t_{i'} + t | s^{ns}) \) for all \( t = 0, 1, \ldots, i = 1, \ldots, n \) and \( c^t_{i'} \in [0, \overline{c}] \).

Let \( D_L(m) \) be the set of equilibrium under the leniency program, satisfying (a)-(d). Likewise let \( D(m) \) be the set of equilibrium without the program, satisfying (a)-(d). Each meaning in the above conditions except (d) is obvious and extra explanation for (d) will follow shortly. The typical example of collusion with above properties is that each player becomes the winner with equal probability, depending on the realization of \( \alpha_t \). Other players than the agreed winner is supposed to bid a little higher than \( m \). If the agreement has been broken at \( t \) for the first time, then everyone switches to the competitive equilibrium forever, i.e. the symmetric Nash equilibrium of the stage auction. Especially, if every member of collusion has the same probability of being the agreed winner at each period, then it is called to be symmetric. Even though, we believe, collusion defined above is quite natural, it is worthwhile to know what other possible ones we are ignoring.

Firstly, the winner and the winning bid in the collusion do not depend on the firms’ cost realization at each period. To maximize the sum of all sellers’ profit, the player with the lowest cost produces the item and the extra profit should be distributed among all the colluders. However to sustain this kind of collusion, asymmetric information problem must
be tackled because each firm’s cost is a private information. This task requires quite a complicated mechanism. The requirement that the agreed winner and the winning bid only depends on the public information helps us not to worry about out-of-equilibrium path belief after any deviation.\footnote{Implementing a complicated collusion requires sophisticated coordination among the colluders. With the risk of being detected, it seems unrealistic for the participants to communicate often to coordinate their behaviors.}

Secondly, a collusion in which a competitive price and a non-competitive high price are circulated along the equilibrium path is excluded. Note that, however, it may have out-of-collusion periods which combine the competitive and non-competitive prices.

Thirdly, (d) states that we don’t consider a collusion where a deviational bidding causes harsher punishment than the competitive bidding. To sustain the collusion, any deviation should be punished in the future and, in principle, the severer punishment will enlarge scope of collusion. However, to justify strong punishment against the deviator such as his min-max strategy, we need to check up the incentive of punishers to participate in the punishment prescribed in the collusion. To avoid this technical complexity, we only consider a collusion in which the bidding in the punishment phase is weakly larger than that of

Lastly, we exclude a collusion with the negative stage game profit by assuming $m > \bar{c}$.

We already described what kind of collusive behavior we impose along the equilibrium path. Because the sequential rationality should on any subgame according to perfect Bayesian Nash equilibrium, we still need to check up several conditions. Firstly, the strategies out of equilibrium path must satisfy sequential rationality of perfect Bayesian Nash Equilibrium. Secondly, the colluders also have incentive to follow the equilibrium of self-reporting strategy. Thirdly, the colluders who are not the agreed winner must have incentive to follow their role. That is, they must not have any incentive to beat the agreed winner by lowering their price.

When we describe a collusion in (a)-(d), we did not specify the colluders’ behavior in the
reporting stage. However, the requirement that the collusion appears along the equilibrium path automatically implies that no self-report should be chosen as long as no deviational bidding has occurred. The intuition is quite simple. If any self-report after \( t \) period bidding is the part of the equilibrium without any previous deviation, there will be no future collusive profit and nobody has incentive to be the agreed non-winner at time \( t \).

**Lemma 1** Pick up any \( d \in D(m) \) or \( d \in D_L(m) \). For any \( t \), if the collusion is maintained up to \( t \), the \( r^t \) prescribes nobody to choose ‘confess.’

**Proof.** Assume to the contrary that \( r^t(h^t; d) \) prescribes seller \( i \) to self-report the collusion along the equilibrium path. Then all the seller will get \( \pi^{ns} \) from \( t + 1 \) period on. Let’s consider the incentive of \( j \neq w^t(h^t; d) \). Since \( b \geq \overline{c} \), \( c_j < m \) with probability 1 and \( j \) has a profitable deviation at time \( t \) which is a contradiction.

If the strategy after any deviation is Nash reversion, not only the reporting strategy along the equilibrium path but also that of out-of-equilibrium path becomes simple. The Nash reversion means that if there has been any deviational bidding, the sellers are supposed to play \( s^{ns} \) at every period. Let \( D^L(m) \) be the set of equilibrium with the Nash reversion punishment under the leniency program, which satisfies (a)-(d). Likewise let \( D(m) \) be the set of equilibrium with the Nash reversion punishment without the program, satisfying (a)-(d). The following Lemma 2 and Lemma 3 state that the equilibrium strategy at the reporting stage after any deviational bidding is quite different under two regimes.

**Lemma 2** Suppose \( d \in D^L(m) \). If the bidding outcome which is different from the agreed one has appeared for the first time at period \( t \), then \( r^t(d) \) must prescribe every colluder to choose ‘confess’.

**Proof.** If the collusion breaks at time \( t \) then from \( t + 1 \) period on, only competitive equilibrium will be played infinitely. Therefore, the average payoff is \( \pi^{ns} \). Then on the\(^8\)
reporting stage, the colluder $i$’s payoff is given by the following table. When the bidders determine whether to confess or not, all the payoff from the previous auction is irrelevant and it is omitted below.

<table>
<thead>
<tr>
<th>At least one other bidder confesses</th>
<th>No other bidders confess</th>
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<tbody>
<tr>
<td>Confess</td>
<td>$(1 - \delta)(-\alpha(k)F - \epsilon) + \delta\pi^{ns}$</td>
</tr>
<tr>
<td>No confess</td>
<td>$(1 - \delta)(-F) + \delta\pi^{ns}$</td>
</tr>
</tbody>
</table>

| Confess                           | $(1 - \delta)(-\alpha(1)F - \epsilon) + \delta\pi^{ns}$ |
| No confess                         | $(1 - \delta)(-\mu F) + \delta\pi^{ns}$ |

Column of the table summarizes the behavior of all other bidders than $i$. If at least one bidder confesses, $i$ gets $(1 - \delta)(-\alpha(k)F - \epsilon) + \delta\pi^{ns}$ by choosing ‘confess’ and $(1 - \delta)(-F) + \delta\pi^{ns}$ by choosing ‘no confess’, where $k$ is the number of colluders who choose confess. The payoffs of the right column represents $i$’s payoff in case that all the other bidders do not report the collusion. $\epsilon < (\mu - \alpha(1))F$ implies that ‘confess’ is the dominant strategy. ■

**Lemma 3** Suppose $d' \in \mathcal{D}(m)$. If the bidding outcome which is different from the agreed one has appeared for the first time at period $t$, then $r^t(d')$ must prescribe every colluders to choose ‘no confess’.

**Proof.** Like the same way as Lemma 2, we can derive the following payoff table and conclude that ‘no confess’ is the dominant strategy without the leniency program.

<table>
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| Confess                           | $(1 - \delta)(-\alpha(1)F - \epsilon) + \delta\pi^{ns}$ |
| No confess                         | $(1 - \delta)(-\mu F) + \delta\pi^{ns}$ |

■
For the statement below, define $m_L, m, m_s^L$ and $m^s$ by

\[
    m_L = \sup \{ m \mid \exists d \in D_L(m) \},
\]
\[
    m = \sup \{ m \mid \exists d \in D(m) \},
\]
\[
    m_s^L = \sup \{ m \mid \exists \text{symmetric } d \in D_L(m) \},
\]
\[
    m^s = \sup \{ m \mid \exists \text{symmetric } d \in D(m) \}. \tag{9}
\]

$m$ is the agreed bidding in the collusion and a bigger $m$ means the bigger profit to the collusion members. However colluders cannot raise $m$ too much. If $m$ is high enough, the agreed non-winners will have big incentive to be the winner this period and willing to cheat other collusion members by bidding $m - \varepsilon$.

The proposition 1, the key result of this paper, shows that the maximum (technically supremum) $m$ which can be achieved through successful collusion under the leniency program is smaller than that of no leniency program case. Hence some collusion which are represented by $m$ between two maxima are not sustainable with the program while they are without program. In this sense, the leniency program can be said to be counter-collusive.\(^{10}\)

The intuition is quiet simple. Without program nobody will self-report after deviational bidding while everybody will resort to it under the program. If $\mu F$, expected fine without the program after deviation, is larger than $\alpha(n)F + \varepsilon$, expected cost with the program, then leniency program makes deviational bidding more profitable, which in turn means more difficult collusion sustainment.

**Proposition 1** Suppose $\mu F > \alpha(n)F + \varepsilon$. Then $m_L < m$.

**Proof.**

\(^{10}\)We can state the same result in terms of discount factor. That is, to sustain a collusion with some given $m$, lower discount factor under the program than without the program is required.
Choose arbitrarily given collusion, \( d \), in \( D_L(m) \). Let \( V_{t+1}^i(d) \) be \( i \)'s continuation payoff from \( t + 1 \) on along the collusion \( d \) if it has not detected until time \( t \). \( \sum_{k=1}^{n} V_{t+1}^k(d) \) does not depend on \( t \) and \( d \), can be denoted by \( \nabla \) and satisfies the following. Note that along the equilibrium nobody will self-report (Lemma 1).

\[
\nabla = (1 - \delta)(m - c^e - n\mu F) + \delta\mu n\pi^{ns} + \delta(1 - \mu)\nabla
\]

\[
\Rightarrow \nabla = \frac{(1 - \delta)}{1 - (1 - \mu)\delta}(m - c^e - n\mu F) + \frac{\delta n\mu \pi^{ns}}{1 - (1 - \mu)\delta}.
\]

where \( c^e \) is the expected production cost.

Let \( j \) be the agreed winner and \( i \) be a non-winner at any given time \( t \). Under the leniency program, \( i \)'s incentive problem is represented by (1). If \( i \) participates in the collusion bidding, his current time expected payoff is \( (1 - \delta)(-\mu F) \) and his/her continuation payoff is \( \mu\pi^{ns} + (1 - \mu)V_{t+1}^i(d) \). In contrast, if \( i \) deviates, the maximum current time payoff he/she can get is \( m \).\(^{11}\) However, because everybody will report the collusion after \( i \)'s deviation (Lemma 2), \( i \) has to pay \( -\alpha(n)F - \varepsilon \) now, the penalty fee and the report cost. In addition, from next period on, only the static Nash equilibrium will be repeatedly played and the continuation payoff is \( \delta\pi^{ns} \). Inequality (2) is the condition that a unilateral confess of player \( i \) to the public authority after no deviation has happened at time \( t \) is not profitable. If \( i \) self-reports, his/her payoff becomes \( (1 - \delta)(-\alpha(1)F - \varepsilon) + \delta\pi^{ns} \) which should be smaller than the equilibrium payoff. The last equality (3) is the condition that the same type of

\(^{11}\)i's profit is \( m \) when the cost is 0 and his winning bid is \( b \).
deviation of the bidder $j$ as (2) is not profitable.\textsuperscript{12}

\[(1 - \delta)(-\mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_i^{t+1}(d)] \geq (1 - \delta)(m - \alpha(n)F - \varepsilon) + \delta \pi^{ns} \quad (1)\]

\[(1 - \delta)(-\mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_i^{t+1}(d)] \geq (1 - \delta)(-\alpha(1)F - \varepsilon) + \delta \pi^{ns} \quad (2)\]

\[(1 - \delta)(m - c_j^t - \mu F) + \delta \left[ \frac{\mu \pi^{ns} + (1 - \mu)V_j^{t+1}(d)}{(1 - \mu)V_j^{t+1}(d)} \right] \geq (1 - \delta) \left( m - c_j^t \right) - \alpha(1)F - \varepsilon) + \delta \pi^{ns} \quad (3)\]

The above inequalities can be arranged as follows.

\[(1 - \mu)V_i^{t+1}(d) \geq \frac{(1 - \delta)}{\delta} (m + \mu F - \alpha(n)F - \varepsilon) + (1 - \mu)\pi^{ns} \quad (4)\]

\[(1 - \mu)V_i^{t+1}(d) \geq \frac{(1 - \delta)}{\delta} (\mu F - \alpha(1)F - \varepsilon) + (1 - \mu)\pi^{ns} \quad (5)\]

\[(1 - \mu)V_j^{t+1}(d) \geq \frac{(1 - \delta)}{\delta} (\mu F - \alpha(1)F - \varepsilon) + (1 - \mu)\pi^{ns} \quad (6)\]

By the same argument, for any $d'$ and $m'$, we need to prove that the followings hold for any $t$ to show $d' \in D_L(m')$.

\[(1 - \mu)V_i^{t+1}(d') \geq \frac{(1 - \delta)}{\delta} m' + (1 - \mu)\pi^{ns},\]

\[(1 - \mu)V_i^{t+1}(d') \geq \frac{(1 - \delta)}{\delta} (\mu F - F) + (1 - \mu)\pi^{ns},\]

\[(1 - \mu)V_j^{t+1}(d') \geq \frac{(1 - \delta)}{\delta} (\mu F - F) + (1 - \mu)\pi^{ns}.\]

(4) and (6) imply that

\[\bar{V} = \sum_{k=1}^{n} V_k^{t+1}(d) \geq \frac{(1 - \delta)}{\delta(1 - \mu)} n \begin{pmatrix} \mu F \\ -\varepsilon \end{pmatrix} + \frac{(1 - \delta)}{\delta(1 - \mu)} n \begin{pmatrix} m - \alpha(n)F \\ -(n - 1)\alpha(1)F \end{pmatrix} + (1 - \mu)n\pi^{ns}.\quad (7)\]

The coefficient of $m$ in the right side of inequality in (7) is $\frac{(1 - \delta)}{1 - \mu}$ and that of $m$ in the

\textsuperscript{12}Because he is the agreed winner at time $t$, we don’t need to check up player $j$’s incentive of equilibrium bidding.
left side is \[\frac{(1-\delta)}{\delta(1-\mu)}, \frac{(1-\delta)(1-\mu)}{1-(1-\mu)\delta} > \frac{(1-\delta)}{\delta}\] because \(\delta > \frac{1}{2(1-\mu)}\). Therefore \(m\) is large enough, then (7) will not hold. In sum, if \(m\) is large enough, any collusion cannot be sustained as an equilibrium. It means that \(\overline{m}\) \((< \infty)\) is well defined. By the same token, we can show \(\overline{m}\) is well defined too.

Let \(d'\) be the collusion without leniency program which has all the same action as \(d\) except two things. First, it has no confess strategy after any non-agreed winner or wining bid has appeared. That is, \(d\) prescribes no colluders to self-report even if the realized winner or the wining bid is different from the agreed one. Second, the agreed wining bid at each period is \(m' \in (m, m + K)\) where \(K \equiv \min\{\mu F - \alpha(n)F - \epsilon, F - \alpha(1)F\}\). The given assumptions guarantee \(K > 0\). By construction, \(V^{t+1}_i(d') > V^{t+1}_i(d)\) and \(V^{t+1}_j(d') > V^{t+1}_j(d)\). Hence, the followings are true from (4)-(6).

\[
(1-\mu)V^{t+1}_i(d') > \frac{(1-\delta)}{\delta}(m + \mu F - \alpha(n)F - \epsilon) + (1-\mu)\pi^{ns} \\
\geq \frac{(1-\delta)}{\delta}(m + K) + (1-\mu)\pi^{ns},
\]

\(8\)

\[
(1-\mu)V^{t+1}_j(d') > \frac{(1-\delta)}{\delta}(\mu F - \alpha(1)F - \epsilon) + (1-\mu)\pi^{ns},
\]

\(9\)

\[
(1-\mu)V^{t+1}_j(d') > \frac{(1-\delta)}{\delta}(\mu - \alpha(1)F - \epsilon) + (1-\mu)\pi^{ns}, \\
\geq \frac{(1-\delta)}{\delta}(\mu - F - \epsilon + K) + (1-\mu)\pi^{ns}.
\]

\(10\)

(8), (9) and (10) conclude that \(d' \in \overline{D}(m')\). Note that the above argument holds for any given \(d \in \overline{D}_L(m)\). In addition, \(K\) is a fixed number without depending on \(t, i, j\) and \(d\). Hence we can say that \(\overline{m} + K \leq \overline{m}\).

Proposition 2 describes the opposite effect of Proposition 1. Specifically, it provides a sufficient condition such that the maximum \(m\) which can be achieved using symmetric collusion without program is smaller than that of leniency program. Hence the leniency program can be said to be pro-collusive in this sense.\(^{13}\) The intuitive understanding of

\(^{13}\) We need symmetry of the collusion to prove Proposition 2, though we believe that the same arguments
the result is that due to the leniency program, self-report to public authority against any
deviation may work as a credible retaliation. The existence of the leniency program may
help punish the deviantional behavior of collusion member and ease collusion formation.

**Proposition 2** Suppose \( \mu F < \alpha(n)F + \varepsilon \). Then \( \overline{m}_L^s > \overline{m}^s \).

**Proof.** Since the other steps are the same as those of Proposition 1, let’s only explain
that the leniency program makes incentive inequalities less demanding. By Lemma 3, under
d a deviantional bidding does not invoke any self-report without the leniency program. Let
d' be the collusion under the leniency which has exactly the same behaviors as d except
that every bidder self-reports after any deviantional bidding.

The conditions of \( d \in \overline{D}(m) \) are given as follows where \( j \) is the agreed winner and \( i \) is
an agreed non-winner at time \( t \).

\[
(1 - \mu)V_i^{t+1}(d) \geq \frac{(1 - \delta)}{\delta} b + (1 - \mu)\pi^{ns}
\]

\[
(1 - \mu)V_j^{t+1}(d) \geq \frac{(1 - \delta)}{\delta} (\mu F - F - \varepsilon) + (1 - \mu)\pi^{ns}
\]

Since \( d \) is symmetric, \( V_j^{t+1}(d) = V_i^{t+1}(d) \). Therefore the above inequalities are reduced
to

\[
(1 - \mu)V_i^{t+1}(d) \geq \frac{(1 - \delta)}{\delta} b + (1 - \mu)\pi^{ns}.
\] (11)

To prove \( d' \in \overline{D}_L(m) \) we have to show that the following inequalities hold.

\[
(1 - \mu)V_i^{t+1}(d') \geq \frac{(1 - \delta)}{\delta} (b + \mu F - \alpha(n)F - \varepsilon) + (1 - \mu)\pi^{ns}
\] (12)

\[
(1 - \mu)V_i^{t+1}(d') \geq \frac{(1 - \delta)}{\delta} (\mu F - \alpha(1)F - \varepsilon) + (1 - \mu)\pi^{ns}
\] (13)

\[
(1 - \mu)V_j^{t+1}(d')] \geq \frac{(1 - \delta)}{\delta} (\mu F - \alpha(1)F - \varepsilon) + (1 - \mu)\pi^{ns}
\] (14)

hold more generally.
\( b > (\alpha(n) - \alpha(1))F \) implies that (13) is true as long as (12) is true. Since \( d' \) is symmetric, \( V_j^{t+1}(d') = V_i^{t+1}(d') \) and (14) is true as long as (13) is true. Therefore the condition of \( d' \in D_L(m) \) boils down to (12). \( \mu F < \alpha(n)F + \varepsilon \) implies that (11) is a stronger condition than (12). ■

Proposition 1 and Proposition 2 provide conditions under which the existence of the leniency program has negative or positive effect on the sustainment of collusion with Nash reversion. The next question is whether these results would hold with more general collusion. For example, consider a collusion in which the retaliation after a deviation is to reverse to Nash equilibrium for a while and then come back to the original collusive bidding. Colluders may not have incentive to confess even with the leniency program after any deviation, because maintaining the prescribed punishment without confess gives the colluders the benefit from future non-competitive bidding.

Proposition 3 and 4 below show that basically the same results as Proposition 1 and 2 can be applied to the collusion mentioned above. The underline argument begins with the observation that the collusion with a finite periods retaliation has weaker punishment power than Nash reversion. Therefore, we can replace the given collusion with the one with more punishment power by introducing the Nash reversion. As before, let \( D_L(m) \) and \( D(m) \) be the set of equilibrium with and without leniency program, respectively, satisfying (a)-(d). Likewise, define \( m_L, m, m^*_L \) and \( m^* \) by

\[
\begin{align*}
m_L &= \sup \{ m \mid \exists d \in D_L(m) \}, \\
m &= \sup \{ m \mid \exists d \in D(m) \}, \\
m^*_L &= \sup \{ m \mid \exists \text{symmetric } d \in D_L(m) \}, \\
m^* &= \sup \{ m \mid \exists \text{symmetric } d \in D(m) \}.
\end{align*}
\]

**Proposition 3** Suppose \( \mu F > \alpha(n)F + \varepsilon \). Then \( m_L < m \).
Proof. The proofs for Proposition 3 and 4 are provided in the appendix.

Proposition 4 Suppose $\mu F < \alpha(n)F + \varepsilon$ and $b > (\alpha(n) - \alpha(1))F$. Then $m^*_L > m^*$.

3 Conclusion

The purpose of this paper is to analyze the effect of the leniency program on collusion. Among two kinds of reduction, one for self-report before investigation and the other for cooperation after investigation, it focuses on the effect of the former.

Specifically, we considered a repeated procurement auction, the most common context of collusion, and added the strategic behavior whether to self-report or not after standard collusive bidding strategies. That is, each colluder simultaneously chooses ‘confess’ or ‘no confess’ after every bidding.

The main conclusion of the paper is that unlike the previous literature which emphasizes no or only negative effect, the leniency program can be pro-collusive. The intuition is that a deviator of a collusion will get less sanction through his/her self-report, which makes the deviation less costly and collusion more difficult. Of course, depending on parameter value, it can be pro-collusive as the previous literature pointed out, because it can play role of a credible threat by non-deviators against deviational behavior.

This result implies the need of careful design of the program to achieve anti-collusive goal.
Appendix

(The proof of Proposition 3) Since all the other steps are the same as the ones in Proposition 1, we only show that the incentive conditions of collusion without the leniency program is weaker than that of the program.

Suppose that $j$ is the agreed winner at time $t$ and $i \neq j$. Because the punishment period payoff may be bigger than that of Nash reversion under $d$, we can not apply Lemma 2 and a deviational bidding may not cause self-report. If $d$ prescribes that $k$ members choose ‘confess’, $d \in D_{L}(m)$ implies (15) holds where $V_{i}^{t+1}(\tilde{\sigma})$ is $i$’ continuation payoff from $t + 1$ on after he/she deviates and no detection by public authority is made at time $t$. On the contrary, if $d$ prescribes that nobody chooses ‘confess’ after any deviational bidding, then (16) must hold.

\[
(1 - \delta)(-\mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_{i}^{t+1}(d)] \geq (1 - \delta)(b - \alpha(k)F - \varepsilon) + \delta[\mu \pi^{ns} + (1 - \mu)V_{i}^{t+1}(\tilde{\sigma})]
\] (15)

\[
(1 - \delta)(-\mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_{i}^{t+1}(d)] \geq (1 - \delta)(b - \mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_{i}^{t+1}(\tilde{\sigma})]
\] (16)

By the assumption $\mu F > \alpha(n)F + \varepsilon$, we know that (16) is more generous condition than (15). Hence, as long as $d \in D_{L}(m)$, at least (16) must hold. In addition, from $i$’s and $j$’th incentive not to self-report at time $t$ in case of no deviational bidding, respectively, we can derive (17) and (18).

\[
(1 - \delta)(-\mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_{i}^{t+1}(d)] \geq (1 - \delta)(-\alpha(n)F - \varepsilon) + \delta \left[ \mu \pi^{ns} + (1 - \mu)V_{i}^{t+1}(\tilde{\sigma}) \right]
\] (17)

\[
(1 - \delta)(-\mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_{j}^{t+1}(d)] \geq (1 - \delta)(-\alpha(1)F - \varepsilon) + \delta \pi^{ns}
\] (18)
Let \( d' \) be the collusion under no leniency program which has exactly the same actions as \( d \) except that it invokes the Nash reversion whenever any deviation has appeared. Note that \( d' \) must prescribe no confess even after a deviational bidding has occurred (Lemma 2). It is also obvious that everybody will follow the equilibrium \( d' \) during the punishment period because it is just the repetition of the static Nash equilibrium. Therefore, To prove \( d' \in D(m) \), we need to show that the following conditions hold.

\[
(1 - \delta)(-\mu F) + \delta \begin{bmatrix} \mu \pi^{ns} \\ +(1 - \mu)V^{t+1}_i(d') \end{bmatrix} \geq (1 - \delta)(b - \mu F) + \delta \begin{bmatrix} \mu \pi^{ns} \\ +(1 - \mu)\pi^{ns} \end{bmatrix}
\]

\[
(1 - \hat{\delta})(-\mu F) + \hat{\delta} \begin{bmatrix} \mu \pi^{ns} \\ +(1 - \mu)V^{t+1}_i(d') \end{bmatrix} \geq (1 - \delta)(-F - \varepsilon) + \hat{\delta} \begin{bmatrix} \mu \pi^{ns} \\ +(1 - \mu)\pi^{ns} \end{bmatrix}
\]

\[
(1 - \hat{\delta})(-\mu F) + \hat{\delta}[\mu \pi^{ns} + (1 - \mu)V^{t+1}_j(d')] \geq (1 - \delta)(-F - \varepsilon) + \hat{\delta}\pi^{ns}
\]

By construction \( V^{t+1}_i(d) = V^{t+1}_i(d') \) and \( V^{t+1}_j(d) = V^{t+1}_j(d') \). Due to (d), \( V^{t+1}_i(\tilde{\sigma}) \geq \pi^{ns} \). Therefore (19) is weaker than (16). In addition, it is obvious that (20) is weaker than (17) and (21) is weaker than (18).

(The proof of Proposition 4) First, note that \( d \) prescribes no confess after any deviational bidding, because (d) means that the payoff over the punishment periods is still larger than Nash payoff and confess only brings the additional cost of \( \varepsilon \). Suppose that up to \( t \), no deviation has occurred. Then \( d \) must satisfy the followings

\[
(1 - \delta)(-\mu F) + \delta \begin{bmatrix} \mu \pi^{ns} \\ +(1 - \mu)V^{t+1}_i(d) \end{bmatrix} \geq (1 - \delta)(b - \mu F) + \delta \begin{bmatrix} \mu \pi^{ns} \\ +(1 - \mu)\pi^{ns} \end{bmatrix}
\]

\[
(1 - \hat{\delta})(-\mu F) + \hat{\delta}[\mu \pi^{ns} + (1 - \mu)V^{t+1}_i(d)] \geq (1 - \hat{\delta})(-F - \varepsilon) + \hat{\delta}\pi^{ns}
\]

\[
(1 - \hat{\delta})(-\mu F) + \hat{\delta}[\mu \pi^{ns} + (1 - \mu)V^{t+1}_j(d)] \geq (1 - \hat{\delta})(-F - \varepsilon) + \hat{\delta}\pi^{ns}
\]
By symmetry of \( d \) and \( V_{t+1}(\tilde{\sigma}) \geq \pi^{ns} \), (22)-(24) boil down to (22).

Let \( d' \) be the collusion under the leniency program which has the exactly same actions as \( d \) except that it invokes Nash Reversion after any deviation has occurred. From Lemma 2, we know that \( d' \) will induce every colluders will self-report after any deviational bidding.

To show \( d' \in D_L(m) \), we need to prove the followings

\[
(1 - \delta)(-\mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_{t+1}^i(d')]) \geq (1 - \delta)(b - \alpha(n)F - \varepsilon) + (1 - \mu)\pi^{ns} (25)
\]

\[
(1 - \delta)(-\mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_{t+1}^i(d')]) \geq (1 - \delta)(-\alpha(1)F - \varepsilon) + (1 - \mu)\pi^{ns} (26)
\]

\[
(1 - \delta)(-\mu F) + \delta[\mu \pi^{ns} + (1 - \mu)V_{t+1}^j(d')]) \geq (1 - \delta)(-\alpha(1)F - \varepsilon) + (1 - \mu)\pi^{ns} (27)
\]

The comparison between (22) and (25)-(27) is exactly the same as one in Proposition 2. Therefore we can conclude that the incentive conditions with under the leniency program are weaker than those without program.
References


