Accounting for a Positive Correlation between Pension and Consumption Taxes

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Abstract. We attempt to account for a puzzling comovement between pension level and the consumption tax rate observed in the OECD data. First, using a standard overlapping generations model with lifetime uncertainty, we can find a set of optimal policy combinations of taxes and pension, but they cannot account for the data. Second, to resolve this puzzle, we consider welfare states where pension level is higher than the optimal level due to external and/or institutional reasons. In this setting, our analysis of optimal tax mix demonstrates that strengthening consumption taxation (relative to income taxation) can improve welfare, i.e., accounting for the proposed puzzle. Third, we also find that population aging further boosts the role of consumption taxation, reinforcing our main findings. Finally, our results lend support to recent pension reforms: when expanding welfare benefits, most countries tend to resort to consumption tax financing.

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I. Introduction

Public pensions usually take the largest share of fiscal expenditures of modern governments, and the recent worldwide trend of population aging reinforces this observation. Accordingly, understanding pensions permits an insight into the fiscal situation of a country. In fact, many developed countries have expanded their welfare programs over time, including public pensions. Currently, this generates a sustainability issue in the face of rising national debt.\(^1\)

Much of the recent discussion has dwelled on a proper reform of public pensions but pension reform has been unsuccessful in many countries around the world. Pension level is largely determined through political processes, and therefore changing the pension level is notoriously difficult although doing so would help the overall society. We will see soon that even among the relatively homogenous OECD countries, there are enough variations in the pension level due to heterogeneity in their political processes. In the era of population aging, pension reform becomes even more difficult, because an increasing elderly population has a greater political voice than ever, adding more rigidity to the institution of pension. Given this, it is an interesting topic to address what the right tax policy should be.

An important empirical motivation behind this topic comes from the data from OECD countries that exhibit a puzzling positive correlation between pension level and consumption tax (see Figure1). This paper attempts to account for this puzzling feature by applying the optimal taxation framework to tax policy structure.

\(^1\) The recent on-going sovereign debt crisis is partly related to excessive government expenditures for redistributive purposes.
Note. The replacement rate is the gross replacement rate of OECD countries, defined as public pension expenditure plus government spending divided by retirees’ income.\(^2\)

To address this puzzle, we examine the role of consumption taxation in an OLG model with lifetime uncertainty. We extend traditional OLG models to allow for the essential role of pension more positively. Unlike traditional pension studies, we permit uncertainty in the absence of complete markets so that public pensions can play the role of providing consumption smoothing insurance to individuals. The lifetime uncertainty here is that not all individuals can live the old-age period, parameterized by survival probability less than unity. Since private insurance against lifetime uncertainty is usually imperfect for adverse selection and moral hazard, i.e., the incomplete market in the Arrow-Debreu sense, public pension retains social insurance function explicitly. We will examine two interesting cases: (i) one is where government can implement both pension and tax policies as choice variables without restrictions, and (ii) the other is where government can implement tax policy without restrictions, but pension level is externally given through political processes.

In our discussion of tax policies, we allow for income and consumption taxes. In an optimal taxation framework, the government maximizes the representative agent’s expected utility, using the tax mix where the labor and capital income taxes are considered along with the consumption tax.\(^3\) Unlike the usual optimal taxation literature (e.g., Atkinson and Stiglitz, 

\(^2\) There are three kinds of pension schemes: public, mandatory private, and voluntary private pension scheme. We take public and mandatory private pensions as a broad class of public pensions since their sum is the level that a society wants to secure and therefore subject to the government’s policy control.

\(^3\) Usually, tax mix refers to an optimal combination between income and consumption taxes. See Lee (2011) for
where the tax mix of direct and indirect taxes cannot be defined for the well-known indeterminacy issue, our framework permits a specific tax mix. Our model can be seen as a modified version of Atkinson and Pestieau’s (1980) two period OLG model, by adding the pension system and lifetime uncertainty.

Using both analytical and quantitative analyses, we show the following. (i) With both pension and tax rates as choice variables of the government, it is impossible to account for the puzzle due to the usual “indeterminacy.” (ii) We can account for the puzzle when pension is determined externally. Only in this case, the consumption tax rate comoves with the pension level. This account for why countries with generous social welfare systems tend to adopt high consumption tax rates. (iii) Through a calibration exercise, we obtain that as population aging advances, strengthening consumption taxation helps social welfare, supporting the recent trend of rising consumption taxes in OECD countries. (iv) These findings can account for why countries expanding their welfare system rely more on consumption taxes.

The intuition behind main results is simple and clear. Given the institutionalized high pension level, we can get closer to the optimal pension level by strengthening consumption taxation so as to lower the real purchasing power of pension. However, consumption taxes cannot go up too high because they also lower the real value of labor income of working population, which is essentially the same with raising the labor income tax. To deal with this, labor income taxes need to be adjusted downward to some extent when consumption tax rates go up in response to an increase in pension benefit. Unlike the usual literature, the capital income tax rate is not necessarily zero in our setting because lifetime uncertainty in our model negatively affects savings incentives. Meanwhile, regarding the relationship between population aging and pension level, the driving force is that as the elderly population becomes larger for aging, young generation’s burden for subsidizing the current pensioners gets larger. Again, given the rigidity against reforming welfare programs, government can only get closer to the optimal level by raising consumption tax rates. At the very least, we

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4 As we will see later, the nature of lifetime uncertainty here is a negative life span shock with the lifetime distribution truncated at the old-age period. This asymmetry in the distribution leads to a negative savings incentive.
believe that this paper provides a new insight into the tax policy: when expenditure-side rigidities are present, a proper tax mix can improve welfare.

This paper is organized as follows. Section II describes our general equilibrium model and presents its key features. Section III presents the indeterminacy issue arising from policy redundancy when government has ability to control all fiscal variables, and next, the optimal tax mix under externally given pension, and the structure of optimal tax mix in the steady state. In Section IV, we build a calibration model, and discuss the policy issues of interest: (a) how the consumption tax rate responds to pension benefit and (b) the population aging effects on the consumption tax and other fiscal variables. The final section summarizes the key results of this paper.

II. The Model

A. Environment

To examine the optimal policy mix of pension and taxes, we construct a standard overlapping generations model with a set of assumptions and conditions as follows. Our model can be seen as extending traditional OLG models (e.g., Atkinson and Pestieau, 1980; Park, 1991) with the new elements of lifetime uncertainty and public pension added.

Agents are identical in terms of both earning ability and preference for consumption and leisure, i.e., representative agent model. This homogeneity assumption deprives pension of a redistributive function within a generation to focus on an intergenerational redistribution dimension. In the qualitative analysis to come, we will see how this supposition affects the optimal labor income taxes. Individuals can live up to two periods, young and old ages, and only young generations can work and old generations live relying on the savings from their youth and public pension without working.

We introduce uncertainty in lifetime such while everyone lives the first period, some individuals do not live through the second period. The probability that an agent live until the second period is defined as survival probability, which can also be interpreted as a degree of population aging in a society. Given this, our model becomes a two periods OLG model with lifetime uncertainty and pension.

Exploiting the neo-classical growth model, firm’s production function is constant
returns to scale (CRS) and a single composite good and factor markets for labor and capital are perfectly competitive.

Government’s primary roles in this model are to collect taxes from both young and old generations and transfer them to old generation in the form of a public pension – there is no government consumption. Public pension acts as an insurance to help smooth consumption in the presence of lifetime uncertainty in an incomplete capital market where agents cannot perfectly cope with their lifetime uncertainty with private saving i.e. self-insurance and private pension systems undergo a market failure because of adverse selection and moral hazard arising from information asymmetry, thus impossible is the complete markets which perfectly insure against the risk.

B. Individual’s Decisions

A representative agent born in period \( t \) can live up to two periods at maximum, \( t \) and \( t + 1 \). With a survival probability \( \theta \) less than unity, agents live the second period \( t + 1 \), i.e., lifetime uncertainty. An individual chooses a single composite good \( C_t^S \) and \( C_t^n \), saving \( a_t \), and labor supply \( l_t \) in perfectly competitive goods and labor markets to maximize the expected value of lifetime utility (1) under constraints (2) and (3):

\[
\max U(C_t^S, C_t^n, Z_{t+1}, l_t) = \theta \cdot \left\{ U^1(C_t^S, l_t) + \beta \cdot U^2(Z_{t+1}) \right\} + (1 - \theta) \cdot \left\{ U^3(C_t^n, l_t) \right\}
\]

s.t. (2) \( (1 + \tau_c, t)C_t^S + a_t = (1 - \tau_w, t)w_t l_t \)

(3) \( (1 + \tau_{c,t+1})Z_{t+1} = (1 + (1 - \tau_r, t+1)r_{t+1}) a_t + m_{t+1} \)

where \( C_t^n = \frac{(1-\tau_w, t)w_t l_t}{(1+\tau_c, t)} \)

\( \theta \): the survival probability;

\( C_t^S \): the first period consumption of those surviving two periods of generation \( t \);

\( C_t^n \): the first period consumption of those surviving one period only of generation \( t \);

\( Z_{t+1} \): the second period consumption of those surviving two periods of generation \( t \);

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5 While \( \theta \) represents lifetime uncertainty in general, a change in \( \theta \) has a bit complicated meaning. When \( \theta \) falls from unity to a number above 0.5, it involves greater randomness and shorter lifespan.
$l_t$: the labor supply of generation $t$;
$a_t$: the savings of generation $t$;
$m_{t+1}$: the pension level in the second period of generation $t$;
$\tau_{ct}$: the consumption tax in period $t$;
$\tau_{c,t+1}$: the consumption tax in period $t + 1$;
$\tau_{w,t}$: the labor income tax in period $t$;
$\tau_{r,t+1}$: the capital income tax in period $t + 1$;
$w_t$: wage rate in period $t$;
$r_{t+1}$: interest rate in period $t$;
$\beta$: time preference;
$U^1(\cdot)$: the first period utility under living two periods;
$U^2(\cdot)$: the second period utility;
$U^3(\cdot)$: the first period utility under living only the first period; and
$t$: the time when variables are observed;

The expected utility is a sum of the lifetime utility when an agent live until the second period and the lifetime utility when an agent live only the first period. Utility function is strictly concave and twice differentiable.

In the first period, agents supply labor $l_t$ and allocate their incomes into the first period consumption $C_t^s$ and $C_t^n$, and savings $a_t$ for the second period consumption – their saving is invested as the next period’s capital stock or is borrowed by a government issuing one period risk free bond. Individuals pay labor income taxes at the rate $\tau_{w,t}$ and consumption taxes at the rate $\tau_{c,t}$ in the first period. All of these are summarized in the first period budget constraint (2).

If individuals survive, they face the second period budget constraint (3), i.e., live on their savings and pension. They pay capital incomes taxes at the rate $\tau_{r,t+1}$ and consumption taxes at the rate $\tau_{c,t+1}$ in the second period. However, individuals’ behaviors in the second period are predetermined because they just consume their savings allocated in the first period.

If individuals do not survive, they are supposed to consume their whole savings.
before dying.\textsuperscript{6} Therefore, the first period consumption in this case, $C_t^n = \frac{(1-\tau_{w,t})w_t l_t}{(1+\tau_{c,t})}$ is different from the first period consumption conditional on survival, $C_t^s$. The value of $C_t^n$ is automatically determined by choosing $l_t$ to maximize the expected value of lifetime utility. Combining two budget constraints (i) and (ii), we can derive a lifetime budget constraint (4). We can rewrite individuals’ optimization under the lifetime budget constraint (4) as follows:

\begin{align*}
(1) \quad & \max U(C_t^s, C_t^n, Z_{t+1}, l_t) = \theta \cdot \{U^1(C_t^s, l_t) + \beta \cdot U^2(Z_{t+1})\} + (1 - \theta) \cdot \{U^3(C_t^n, l_t)\} \\
\text{s.t.} \quad & (1 + \tau_{c,t})C_t^s + \frac{(1+\tau_{c,t+1})Z_{t+1}}{(1+(1-\tau_{c,t+1})r_{t+1})} = (1 - \tau_{w,t})w_t l_t + \frac{m_{t+1}}{(1+(1-\tau_{c,t+1})r_{t+1})}
\end{align*}

where $C_t^n = \frac{(1-\tau_{w,t})w_t l_t}{(1+\tau_{c,t})}$

In this setting, agents choose $Z_{t+1}$ instead of $a_t$ to maximize the expected value of lifetime utility (1) under the lifetime budget constraint (4).

Now, individual optimization gives two Euler equations as follows:

\begin{align*}
(5) \quad & (\theta \cdot U^1_{c,t} + (1 - \theta) \cdot U^3_{c,t}) \frac{(1-\tau_{w,t})w_t}{(1+\tau_{c,t})} = -(\theta \cdot U^1_{l,t} + (1 - \theta) \cdot U^3_{l,t}) \\
(6) \quad & \beta \cdot U^2_{Z_{t+1}} - \frac{\theta \cdot U^1_{c,t}}{(1+\tau_{c,t})} \cdot \frac{(1+\tau_{c,t+1})}{(1+(1-\tau_{c,t+1})r_{t+1})} = 0
\end{align*}

Equation (5) indicates the intra-temporal choice between current consumption and labor. (6) shows the inter-temporal choices between current and future consumption.\textsuperscript{7}

C. Firm’s Decisions

Firms adopt a constant returns to scale (CRS) production technology. The production function in period $t$ can be changed into per capita version:

\begin{align*}
(7) \quad & F(K_t, L_t^d) \rightarrow F(k_t, l_t^d)
\end{align*}

\textsuperscript{6} As in usual OLG models, agents in our model can only consume their savings, not leaving bequest to the next generation before dying. Of course, we can take a different modeling strategy such that the assets those not surviving the second period leave behind are taken by the government for managing public pensions. Since the main conclusion does not change in either case, we will adopt the current assumption.

\textsuperscript{7} Derivations of two Euler equations are presented in Appendix A.
where $K_t$: the capital stock in period $t$; $L_t^d$: the labor demand in period $t$; $k_t$: the capital stock per capita in period $t$; $l_t^d$: the labor demand per capita in period $t$

A firm chooses capital stock per capita $k_t$ and labor demand per capita $l_t^d$ in perfectly competitive capital and labor markets to maximize profit $\pi_t$:

$\max \pi_t = [\mathcal{F}(k_t, l_t^d) - w_t l_t^d - \gamma_t k_t]$, $\gamma_t = r_t$

where $w_t$: wage rate in period $t$; $\gamma_t$: rental rate in period $t$; $r_t$: interest rate in period $t$;

The rental rate $\gamma_t$ is equivalent to interest rate $r_t$. Firms’ optimization behavior creates two first-order conditions (9). One determines wage rate $w_t$ and the other decides rental rate $\gamma_t$:

$w_t = F_l(k_t, l_t^d)$, $\gamma_t = F_k(k_t, l_t^d)$

Marginal products of labor and capital are equal to wage and rental rates, respectively, at the optimum.

D. Factor Market Conditions

Given competitive factor markets, the labor market equilibrium in period $t$ occurs when demand for labor by firms equal to supply of labor:

$l_t^d = l_t$

In the capital market, the next period’s capital stock per capita is equal to private saving minus national debt in period $t$, which derives a capital market condition:

$(1 + n)k_{t+1} = \theta \cdot a_t - b_t$

where $b_t$: the per capita national debt issued in period $t$.

Again, population growth and lifetime uncertainty modifies the conventional capital market equation only slightly. We can replace private saving $a_t$ in (11) with an individual’s first period budget constraint (2) to derive a new capital market condition:

$(1 + n)k_{t+1} = \theta \cdot \{(1 - \tau_{w,t})w_t l_t - \tau_{e,t} C_t^s\} - b_t$
E. Resource Constraint

The total resource of an economy in period $t$, the sum of output and the capital stock in that period, is consumed by the young generation born in period $t$ and the old generation born in period $t-1$ who survives in period $t$ and the rest turns into the capital stock for the next period, $t+1$:

$F(k_t, l_t) + k_t = \theta \cdot C_t^s + (1 - \theta) \cdot C_t^n + \frac{\theta}{1+n} Z_t + (1+n)k_{t+1}$

where $C_t^n = \frac{(1-\tau_{w,t}) w_t l_t}{(1+\tau_{c,t})}$

Note that the current consumption comes from young and old generations, and among the young, $\theta$ proportion of them consumes $C_t^s$, $(1-\theta)$ proportion of them consumes $C_t^n$, and the old generation is fewer than young generation as reflected by the factor $\frac{\theta}{1+n}$ for population growth and lifetime uncertainty.

F. Government Budget Constraint

The total government revenues in period $t$ come from newly issuing one-period risk free bond and linear tax system and they are transferred in the form of public pension to the old generation born in period $t-1$ who survive in period $t$ and the rest is used for the repayment of the existing bond. The government budget constraint in period $t$ is:

$\left(\tau_{w,t} w_t l_t + \tau_{c,t} \left(\theta \cdot C_t^s + (1 - \theta) \cdot C_t^n\right)\right) + \frac{1}{1+n} \left(\theta \cdot \tau_{r,t} r_t a_{t-1} + \theta \cdot \tau_{c,t} Z_t\right)$

$= \frac{1 + r_t}{1 + n} b_{t-1} + \frac{\theta}{1 + n} m_t$

where $C_t^n = \frac{(1-\tau_{w,t}) w_t l_t}{(1+\tau_{c,t})}$, $b_t$ is the per capita national debt issued in period $t$.

Variables pertaining to old generation are discounted by the population growth rate $n$ and life
time uncertainty parameter $\theta$ applies to the surviving old population.\textsuperscript{8} The existing bond, i.e., national debt in the previous period, is multiplied by $\frac{1+r_t}{1+n}$ because of paying interest and population growth.

G. Equilibrium

Given factor prices $w_t$ and $y_t$ and fiscal policy variables $\tau_{c,t}, \tau_{w,t}, \tau_{r,t}, b_t$ and $m_t$ in period $t$, the representative individual chooses $c_t^s, c_t^n, Z_t$ and $l_t$ to maximize the expected value of lifetime utility, and firms choose $k_t$ and $l_t^d$ to maximize the profit. If the resulting feasible allocation $c_t^s, c_t^n, Z_t, k_t, l_t$ and $l_t^d$ and a set of prices satisfy the aforementioned capital and the labor market condition, and the resource constraint, and fiscal policies $\tau_{c,t}, \tau_{w,t}, \tau_{r,t}, b_t$ and $m_t$ satisfy the government budget constraint, we obtain a competitive equilibrium in period $t$.

III. The Ramsey Problem

In this section, we discuss the optimal fiscal policy in a decentralized competitive economy. To deal with this topic, we construct a Ramsey problem as follows:

Government can chooses fiscal policy instruments to maximize the representative agent’s expected utility in four cases: (i) government can implement both pension and tax policies as choice variables without restrictions, (ii) government can implement tax policy without restrictions, but pension level is externally given through political processes. (iii) government can implement tax policy without restrictions in response to a variation of externally determined pension level and (iv) government can implement tax policy without restrictions in response to a variation of a degree of population aging with fixed pension benefit.

We will examine what the optimal fiscal policy should be in each case i.e. solving each Ramsey problem.

\textsuperscript{8} For instance, a factor $\frac{\theta}{1+n}$ is used to represent that old generations are this much fewer than young generation because of population growth and lifetime uncertainty.
A. Indeterminacy issue

In this two period OLG model with lifetime uncertainty and pension, government cannot uniquely determine all five fiscal policy instruments $\tau_{c,t}$, $\tau_{w,t}$, $\tau_{r,t}$, $b_t$ and $m_t$, i.e., the usual indeterminacy problem. Although we set the bond $b_t$ as an external variable as in the literature on the optimal fiscal policy, this indeterminacy still arises. The nature of indeterminacy lies in that a set of fiscal policy instruments can achieve the same welfare. We summarize this point using Proposition 1.

**Proposition 1. The indeterminacy.** It is impossible to determine the optimal rates of income and consumption taxes along with the optimal pension level.

**Proof.** Normalize the price of the future consumption in the individual’s lifetime budget constraint (4) first. We can obtain a new individual budget constraint (IBC) (16) below:

$$P_t'c_t^s + Z_{t+1} = \omega_t'l_t + m_{t+1}'$$

where $P_t' = \{(1 + (1 - \tau_{r,t+1})r_{t+1})(1 + \tau_{c,t+1})\}/(1 + \tau_{c,t+1})$, $\omega_t' = \{(1 - \tau_{w,t})(1 + (1 - \tau_{r,t+1})r_{t+1})\} \cdot w_t/(1 + \tau_{c,t+1})$ and $m_{t+1}' = m_{t+1}/(1 + \tau_{c,t+1})$.

Three elements needed to achieve social welfare maximization in (16) are three: (i) the relative price between current and future consumption, $P_t'$, (ii) the relative price between future consumption and labor, $\omega_t'$, and (iii) the real value of pension benefit, $m_{t+1}'$. However, government has four fiscal policy instruments $\tau_{c,t}$, $\tau_{w,t}$, $\tau_{r,t}$ and $m_t$ that affect two relative prices and one real value of pension benefit simultaneously. Note that national debt, $b_t$, is predetermined to satisfy the GBC. Therefore, given this policy redundancy, there exists a set of fiscal policy instruments leading to an identical level of social welfare.

For this reason, it is impossible to uniquely determine the optimal level of all policy instruments, i.e., there is no unique Ramsey allocation.

B. Optimal tax mix

With all five fiscal policy instruments $\tau_{c,t}$, $\tau_{w,t}$, $\tau_{r,t}$, $b_t$ and $m_t$ as choice variables of a government, policy indeterminacy arises. However, if pension level is externally given, it is possible to find unique set of fiscal policy instruments, i.e., there exists a unique Ramsey allocation.

**Proposition 2. The optimal tax mix.** With an externally given pension level, it is possible to determine the optimal rates of income and consumption taxes under the uniformity of consumption tax rate across periods.
Proof. Going back to the new IBC (16) again, if pension level \( m_t \) is externally determined, government have three fiscal policy instruments \( \tau_{c,t}, \tau_{w,t}, \) and \( \tau_{r,t} \) that affect two relative prices and one real value of pension benefit simultaneously where national debt, \( b_t \), is predetermined to satisfy the GBC. Therefore, we can uniquely determine the optimal tax rates.

Difficulties of pension reform in many modern developed economies support the condition that pension level \( m_t \) is externally determined. It is not very surprising because a generous social security system is one of underlying principles of welfare states.

C. The structure of optimal tax mix in the steady state

In this subsection, we discuss further the structure of optimal tax mix in the steady state to see if the pattern of optimal tax mix combined with pension can account for the puzzle observed in the OECD data using analytical approach. To find the steady state optimal tax mix with exogenously given pension in this dynamic problem, we construct the social planner’s problem using a Bellman equation. To construct the Ramsey problem, we need to modify IBC, RC, capital and labor market conditions as follows:

\[
\text{Individual’s decisions under modified individual budget constraint}
\]

\[
\begin{align*}
\max & \ U(C_t^S, C_t^n, Z_{t+1}, l_t) = \theta \cdot \left\{ U^1(C_t^S, l_t) + \beta \cdot U^2(Z_{t+1}) \right\} + (1 - \theta) \cdot \left\{ U^3(C_t^n, l_t) \right\} \\
\text{subject to} & \quad P_tC_t^S + (1 + \tau_{c,t+1})Z_{t+1} = \omega_t l_t + m_{t+1},
\end{align*}
\]

where \( C_t^n = \frac{(1 - \tau_{w,t})w_t l_t}{(1 + \tau_{c,t})}, P_t = (1 + (1 - \tau_{r,t+1})r_{t+1})(1 + \tau_{c,t}), \) \( \omega_t = (1 - \tau_{w,t})(1 + (1 - \tau_{r,t+1})r_{t+1}) \cdot w_t. \)

In the modified IBC (17), the price of the current consumption when surviving and labor the wage substituted with \( P_t \) and \( \omega_t \), respectively. Thus, the maximized utility, i.e., indirect utility and the optimal value of control variables resulted from the optimizing process are represented by such a new price system and pension level as follows:

\[
\begin{align*}
\text{(18-1) } & \quad X_t = X_t(P_t, \tau_{c,t+1}, \omega_t, m_{t+1}; \theta, \beta), \text{ for } X = C^n, Z, l \\
\text{(18-2) } & \quad V_t = V_t(P_t, \tau_{c,t+1}, \omega_t, m_{t+1}; \theta, \beta)
\end{align*}
\]
Combining RC, Capital and Labor market equilibrium conditions

To reduce the number of equations which have to be considered, we combine RC, capital and labor market equilibrium conditions in the following equilibrium conditions now:

\[
F(k_t, l_t) + k_t + \theta \cdot C_{c,t}^s - (1 - \theta) \cdot C_t^n - \frac{\theta}{1+n} Z_t - \theta \cdot (1 - \tau_{w,t}) w_t l_t + b_t = 0
\]

where \( C_t^n = \frac{(1-\tau_{w,t}) w_t l_t}{(1+\tau_{c,t})} \)

Social welfare function

The social welfare function in period \( t \) is comprised of indirect utilities from both \( t \) generation and the following generations who are not yet born:

\[
W_t(k_t, \tau_{r,t}, \tau_{c,t}, \tau_{w,t}, m_t) = \max \sum_{j=t}^{\infty} (\frac{1}{1+\rho})^{j-t} \cdot V_j
\]

where \( \rho \) is the social discount rate.

Instead of using the new price system, \( P_t, \tau_{c,t+1} \), and \( \omega_t \), we express the social welfare function in period \( t \) using state variables - capital stock, tax rates, and pension level - to determine optimal allocations directly.

Social planner’s problem

To construct social planner’s problem, we need to consider five constraints IBC, RC, GBC, capital and labor market equilibrium conditions in maximizing the social welfare. However, we can eliminate GBC according to Walras’ law. IBC is reflected in the indirect utility. Therefore, we can consider the compact expression (19) only in the social planner’s problem. This is summarized in (21) using a dynamic program so called Bellman equation:

\[
W_t(k_t, \tau_{r,t}, \tau_{c,t}, \tau_{w,t-1}, m_t) = \max \left[ V_t + \mu_t \left\{ F(k_t, l_t) + k_t - \theta \cdot C_t^s - (1 - \theta) \cdot C_t^n - \frac{\theta}{1+n} Z_t - (1 + n)k_{t+1} \right\} \right.
\]

where \( C_t^n = \frac{(1-\tau_{w,t}) w_t l_t}{(1+\tau_{c,t})} \), \( \mu_t \) = the Lagrange multiplier for RC.
Optimal tax structure in steady state

The FOCs of the social planner’s problem (21) with respect to tax rates \(\tau_{r,t+1}, \tau_{c,t+1}, \tau_{w,t}\) are as follows:

\[
(22-1) \quad \frac{\partial v_t}{\partial \tau_{r,t+1}} = \mu_t \left[ (w_t l_{t,p} - C_{t,p}^c)P_{t,r,t+1} + (w_t l_{t,w} - C_{t,w}^c)\omega_{t,r,t+1} \right] + \frac{1}{1+\rho} W_2(t + 1)
\]

\[
(22-2) \quad \frac{\partial v_t}{\partial \tau_{c,t+1}} = \mu_t \left[ (w_t l_{t,c,t+1} - C_{t,c,t+1}^c) + (w_t l_{t,p} - C_{t,p}^c)P_{t,c,t+1} + (w_t l_{t,w} - C_{t,w}^c)\omega_{t,c,t+1} \right] + \frac{1}{1+\rho} W_3(t + 1)
\]

\[
(22-3) \quad \frac{\partial v_t}{\partial \tau_{w,t}} = \mu_t \left[ (w_t l_{t,p} - C_{t,p}^c)P_{t,w,t} + (w_t l_{t,w} - C_{t,w}^c)\omega_{t,w,t} \right] + \frac{1}{1+\rho} W_4(t + 1)
\]

where \(C_t^c = \theta \cdot C_t^s - (1 - \theta) \cdot C_t^n\), \(\mu_t = W_1(t + 1) / ((1 + n)(1 + \rho))\).

The Lagrange multiplier of RC is equal to \(W_1(t + 1) / ((1 + n)(1 + \rho))\). This indicates that the social marginal utility of unit capital at the current period is equivalent to the present value of social marginal utility of \(\frac{1}{1+n}\) capital in the next period at the optimum. The intuitive meaning of the expression is that the cost of investing unit capital for the next period should be equal to the benefit of it.

The differentiated value function series \(W_t\) with respect to tax rates and capital stock, \(\tau_{r,t}, \tau_{c,t}, \tau_{w,t-1}\) and \(k_t\), using the Benveniste-Scheinkman formula are as follows:

\[
(23-1) \quad \frac{\partial v_t}{\partial \tau_{r,t}}: W_2(t) = -\frac{\theta}{1+n} \mu_t \left[ Z_{t,p} P_{t,r,t} + Z_{t,w} \omega_{t,r,t} \right]
\]

\[
(23-2) \quad \frac{\partial v_t}{\partial \tau_{c,t}}: W_3(t) = -\frac{\theta}{1+n} \mu_t \left[ Z_{t,c,t} + Z_{t,p} P_{t,c,t} + Z_{t,w} \omega_{t,c,t} \right] + \mu_t \left[ (w_t l_{t,p} - C_{t,p}^c)P_{t,c,t} + (w_t l_{t,w} - C_{t,w}^c)\omega_{t,c,t} \right]
\]

\[
(23-3) \quad \frac{\partial v_t}{\partial \tau_{w,t-1}}: W_4(t) = -\frac{\theta}{1+n} \mu_t \left[ Z_{t,p} P_{t,w,t-1} + Z_{t,w} \omega_{t,w,t-1} \right]
\]

\[
(23-4) \quad \frac{\partial v_t}{\partial k_t}: [W_4(t) - \mu_t(1 + r)] = \frac{\theta}{(1+n)(1+\rho)} \left[ Z_{t,p} P_{k,t} + Z_{t,w} \omega_{k,t} \right]
\]

where \(C_t^c = \theta \cdot C_t^s - (1 - \theta) \cdot C_t^n\).
Using Roy’s identities, we can express the partial derivatives of indirect utilities with respect to $\tau_{r,t+1}, \tau_{c,t+1}, \tau_{w,t}$ and $k_t$ as follows:

\[ \frac{\partial V_t}{\partial \tau_{r,t+1}} = -\lambda_t \left[ C_t^s P_{\tau_{r,t+1}} - l_t \omega_{\tau_{r,t+1}} \right] \]

\[ \frac{\partial V_t}{\partial \tau_{c,t+1}} = -\lambda_t \left[ C_t^s P_{\tau_{c,t+1}} + Z_t - l_t \omega_{\tau_{c,t+1}} \right] \]

\[ \frac{\partial V_t}{\partial \tau_{w,t}} = -\lambda_t \left[ C_t^s P_{\tau_{w,t}} - l_t \omega_{\tau_{w,t}} \right] \]

\[ \frac{\partial V_t}{\partial k_t} = -\lambda_t \left[ C_t^s P_{k_t} - l_t \omega_{k_t} \right] \]

where $\lambda_t$ is the marginal private utility of income.

By substituting the differentiated value function series $W_t$ in (23) and Roy’s identity result (24) into the steady-state version of the FOCs in (22), we obtain the following homogeneous equation system in the steady state:

\[ A_1 P_{\tau_r} - A_3 \omega_{\tau_r} = 0 \]

\[ A_1 P_{\tau_c} + A_2 - A_3 \omega_{\tau_c} + \frac{\mu}{1+\rho} \left[ (wl_p - C_{\mu}^c)P_{\tau_c} + (wl_\omega - C_{\omega}^c)\omega_{\tau_c} \right] = 0 \]

\[ A_1 P_{\tau_w} - A_3 \omega_{\tau_w} = 0 \]

\[ [W_1(t) - \mu_t(1+r)] = \mu[(1+n)(1+\rho) - (1+r)] = A_1 P_k - A_3 \omega_k \]

Here, we ignore the time subscript because we focus on the steady-state. The only solution to this homogeneous-equation system is as follows:

\[ A_1 = \lambda C^s - \mu (wl_p - C_{\rho}^c + (K - (1+\tau_c)) \cdot \theta Z_p) = 0 \]

\[ A_2 = \lambda Z - \mu (wl_{\tau_c} - C_{\tau_c}^c + (K - (1+\tau_c)) \cdot \theta Z_{\tau_c}) = 0 \]

\[ A_3 = \lambda l - \mu (C_{\omega}^c - wl_\omega - (K - (1+\tau_c)) \cdot \theta Z_\omega) = 0 \]

\[ \frac{\mu}{1+\rho} \left[ (wl_p - C_{\mu}^c)P_{\tau_c} + (wl_\omega - C_{\omega}^c)\omega_{\tau_c} \right] = 0 \]

\[ (1+n)(1+\rho) = (1+r) \]
where \( C^c = \theta \cdot C^s - (1 - \theta) \cdot C^n, \quad K = (1 + \tau_c) - \frac{1}{(1+n)(1+p)} \)

The last equation (26-5) is often referred to as the modified golden rule which determines the optimal capital stock per capita. This result indicates that the capital stock is set at its first-best level. Using the partial derivatives of the steady-state version of the modified IBC (17), we can rewrite the \( A_i \) for \( i = 1, 2 \) and 3 as follows:

(27-1) \( A_1 = \lambda C^s - \mu(C^s \theta + C^p \theta (P - 1) \theta - (1 - \theta) \cdot C^n + Z_p K \theta + l_p(w - \theta \omega)) = 0 \)

(27-2) \( A_2 = \lambda Z - \mu \left( Z \theta + C_{\tau \epsilon} \frac{Z}{\tau \epsilon} (P - 1) \theta - (1 - \theta) \cdot C_{\tau \epsilon} + Z_{\tau \epsilon} K \theta + l_{\tau \epsilon}(w - \theta \omega) \right) = 0 \)

(27-3) \( A_3 = \lambda l - \mu(l \theta - C_n \frac{1}{\alpha}(P - 1) \theta + (1 - \theta) \cdot C_n - Z_n K \theta - l_n(w - \theta \omega)) = 0 \)

Note that the terms \( C_p \), \( Z_p \) and \( l_p \) denote the direct marginal (partial) effect of a change in the rate of \( P \), not the total marginal effect. Using the properties of the Slutsky equations, \( A_i \) for \( i = 1, 2 \) and 3 (27) can be simplified in a matrix form:

(28) \[
\begin{bmatrix}
S_{C_p} & S_{Z_p} & S_{l_p} \\
S_{\tau \epsilon} & S_{\tau \epsilon} & S_{l_{\tau \epsilon}} \\
-S_{\omega} & -S_{\omega} & -S_{l_{\omega}}
\end{bmatrix}
\begin{bmatrix}
(P - 1) \theta \\
K \theta \\
w - \omega \theta
\end{bmatrix}
=
\begin{bmatrix}
\left( \frac{\lambda}{\mu} - \theta \right) C^s + (1 - \theta) \cdot C^n \\
\left( \frac{\lambda}{\mu} - \theta \right) Z + (1 - \theta) \cdot C_{\tau \epsilon} \\
\left( \frac{\lambda}{\mu} - \theta \right) l - (1 - \theta) \cdot C_n
\end{bmatrix}
\]

where \[
\begin{bmatrix}
S_{C_p} & S_{Z_p} & S_{l_p} \\
S_{\tau \epsilon} & S_{\tau \epsilon} & S_{l_{\tau \epsilon}} \\
-S_{\omega} & -S_{\omega} & -S_{l_{\omega}}
\end{bmatrix}
= \begin{bmatrix}
C_p & Z_p & l_p \\
C_{\tau \epsilon} & Z_{\tau \epsilon} & l_{\tau \epsilon} \\
-C_n & -Z_n & -l_n
\end{bmatrix}.
\]

The coefficient matrix in the LHS is a matrix of substitution terms which is negative semi-definite and symmetric. The Slutsky terms shows the following properties. (i) \( S_{Z_p} = S_{\tau \epsilon} \), \( S_{l_p} = -S_{\omega} \), \( S_{l_{\tau \epsilon}} = -S_{l_{\omega}} \) and (ii) the only two terms \( S_{Z_p}, S_{\tau \epsilon} \) are positive and the all other terms are negative. The first principal sub-matrix of the coefficient matrix of (28) is negative because \( S_{C_p} < 0 \). The second principal sub-matrix is positive to make the coefficient matrix negative semi-definite. We verify that the third principal sub-matrix is non-positive, but not that it is negative. With the assumption that the determinant of the coefficient matrix is not zero, we can be sure that the third principal sub-matrix is negative and then, the coefficient matrix is negative definite. In this case, the coefficient matrix is non-singular, so the optimal tax mix in the steady can be determined as follows:

(29) \( (P - 1) \theta = \frac{D_1}{D}, K \theta = \frac{D_2}{D}, w - \omega \theta = \frac{D_3}{D} \)

where \( D \) denotes the determinant of the coefficient matrix in the LHS of (28) and \( D_1 \) denotes the determinant of the coefficient matrix with the first column replaced by the matrix
on the RHS of (28), and similar notations apply to $D_2$ and $D_3$. By substituting $\tau_r, \tau_c$ and $\tau_w$ into $P, K$ and $\omega$ of equation (29), we can find the structure of optimal tax mix in steady-state, $\tau_r^*, \tau_c^*$ and $\tau_w^*$, represented by $D, D_1, D_2, D_3$ and $\theta$.

### IV. Quantitative Analysis

In this section, using a calibrated model, we will conduct some policy experiments based on the earlier analytical results. Primarily, we focus on accounting for a positive correlation between pension and consumption tax rate.

**A. Data**

Based on our numerical model with a few specifications, we examine whether the predictions derived in the theory section really apply to the OECD countries’ data, and we try to find other interesting implications which cannot be addressed in the theory part due to the lack of analytical tractability arising from nonlinear equations. Among the OECD countries, we think that there is sufficient enough variation in the pension level due to heterogeneity in their political processes. We use information from OECD about fiscal policies including tax structure and pension benefits. Given difficulties of obtaining mortality rate at each age from each of the OECD countries, we use the comparable information from Statistics Korea as an alternative.

**B. Our Numerical Model**

This section presents a numerical model and calibrates our model to fit the U.S. data. We introduce the following specifications. First, the preferences are depicted by Kimball and Shapiro’s (2008) form of the King-Plosser-Rebelo (KPR) utility function which is widely exploited in macro economics literatures:

\[
U(C, l) = \frac{c^{1-\gamma}}{1-\gamma} \left[ 1 + M \left( 1 - \frac{1}{\gamma} \right)^{\frac{1+\eta}{1+\gamma}} \right]^\gamma, \text{with } \gamma > 0, M > 0, \eta > 0
\]

where $\gamma$ is the relative risk aversion, $M$ is the work aversion parameter, and $\eta$ the effort supply elasticity. The utility function is not separable between consumption and labor, so our model is in a more general setting.
Second, the production technology is described by Cobb-Douglas production function:

\[(31) \quad F(k, l) = A k^{\alpha} l^{1-\alpha}, \text{ with } \alpha > 0, A > 0\]

where \(A\): the total factor productivity, and \(\alpha\): the capital share in production.

We set calibration parameter values such that our OLG model fits the real economy as closely as possible. The first period in this model is equivalent to the working period (age 25 to 60) in reality and the second period is to the non-working period (age 60 to 95).

Calibration parameter values are set as follows (see Table 1). As the benchmark relative risk aversion and elasticity of labor supply, we set \(\gamma = 2\) and \(\eta = 0.5\). The relative risk aversion value is popularly used in many empirical literatures. Whereas the labor supply elasticity can have a wide range of value, we borrow the estimate from Lee (2001; 2008). We normalize total factor productivity \(A\) as 1. The time preference \(\beta\) is 0.4. The widely used time preference for one year is between 0.96 and 0.99, but the one period of this model corresponds to 35 years in the real economy. Thus, we set \(\beta = 0.975^{35} \approx 0.4\).

Some parameter values are selected to be consistent with the real data: the survival probability \(\theta\) and population growth rate \(n\) for OECD countries are 0.81\(^9\) and 0.006, respectively. Other parameter values are chosen to help fit the OECD data: (a) the discount rate \(\rho = 0.009\), and the work aversion parameter \(M = 0.65\) are set to fit the observed OECD data – a pension replacement rate of 57.0\%, and the real interest rate of 1.5\%\(^{10}\). To determine the exogenously given pension level, we have to fix one of other policy instruments due to the indeterminacy issue. Setting the consumption tax rate at a benchmark rate of 0.185, i.e., the average rate of OECD countries, we can find an optimal pension level, \(m = 4.966\). Later, we will use a value greater than this optimal level to denote an excessively

---

\(^9\) The survival probability from age 15 to 60 is 0.92 which is an estimate from the OECD countries data. From this, the survival probability for the first period in our model (i.e., age 25 to 60) can be calculated to be 0.94. However, all of those who survive after the working period do not live until the end of the second, non-working period. To fill the gap between the data and our model, we calibrate \(\theta\) as follows:

\[
\theta = 0.94 \times \left(\frac{\sum (\text{the number of years each agent lives during non-working period} \times \text{weight})}{35 \times \text{the total number of agents who live at least a year during non-working period}}\right).
\]

Here, the weight takes into account the interest rate because the real values of pension entitlement of different times during the non-working period are different. Using the life table from Statistics Korea, we calibrate the survival probability to be 0.81.

\(^{10}\) The consumption tax rate here is the average of those in all OECD countries. Pension replacement ratio refers to gross pension replacement rate which includes both public pension and mandatory private pension. The real interest rate is defined as the long term interest rate of ten-year treasury bond minus inflation rate. All data are collected in year 2011.
high pension.

Table 1. The Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>The relative risk aversion</td>
<td>2.000</td>
</tr>
<tr>
<td>$\eta$</td>
<td>The effort supply elasticity</td>
<td>0.500</td>
</tr>
<tr>
<td>$M$</td>
<td>The work aversion parameter</td>
<td>0.650</td>
</tr>
<tr>
<td>$A$</td>
<td>Total Factor Productivity</td>
<td>1.000</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital Share in Production</td>
<td>0.330</td>
</tr>
<tr>
<td>$m$</td>
<td>Given pension level</td>
<td>0.431</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Survival probability</td>
<td>0.810</td>
</tr>
<tr>
<td>$n$</td>
<td>Population growth rate</td>
<td>0.006</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Social discount rate</td>
<td>0.717</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Time Preference</td>
<td>0.700</td>
</tr>
</tbody>
</table>

C. Simulation Results

*The Base Case Numerical Results.* Under the set of calibrated parameters value, we obtain the steady-state Ramsey allocations which maximize the social welfare (see Table 2). The optimal equilibrium on key variables is as follows: \( \tau_c = 0.185, \tau_w = -0.179, \tau_r = -0.171 \). Putting a number of different initial values on our numerical model, the same numerical result is generated. This result confirms Proposition 2 that the unique optimal rates of income and consumption taxes are identified.

Table 2. The Base Case Results

<table>
<thead>
<tr>
<th>variable</th>
<th>$\tau_c$</th>
<th>$\tau_w$</th>
<th>$\tau_r$</th>
<th>$C$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>variable</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>l</td>
<td>1.265</td>
<td>0.055</td>
<td>0.420</td>
<td>0.389</td>
<td>0.727</td>
</tr>
</tbody>
</table>

Note. $RR$: replacement ratio = present value of pension benefit / income = \( (1 + (1 - \tau_r) \cdot r)^{-1} \).
The labor income tax here is unrealistically lower than the usual rate observed in the real economy. This is in fact unsurprising because we do not allow ability difference across individuals. Next, the capital income tax rate is in fact positive in modern economies. However, the tax rate is strangely negative in this numerical model. This phenomenon also can be elucidated by our assumptions to make model simple, but I will deal with it in part (e) of this section.

**Comparative Statics Analyses.** We present results from comparative statics analysis to show how optimal fiscal policy and Ramsey allocations respond to the changes in pension benefit and parameter values in the steady state (see Table 3).

In this subsection, we increase model parameters including pension benefit by 10%. For survival probability \( \theta \), we run experiment by increasing the survival probability by 0.03 to keep it less than unity. Considering both ‘incentive’ and ‘income guarantee’ effects of pension, social planner chooses the optimal tax mix differently in response to the changes in parameter values. The comparative statics results are shown in Table 3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Base Case</th>
<th>↑m</th>
<th>↑n</th>
<th>↑( \theta )</th>
<th>↑y</th>
<th>↑M</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_c )</td>
<td>0.185</td>
<td>0.303</td>
<td>0.186</td>
<td>0.393</td>
<td>0.175</td>
<td>0.223</td>
</tr>
<tr>
<td>( \tau_w )</td>
<td>-0.099</td>
<td>-0.209</td>
<td>-0.100</td>
<td>-0.308</td>
<td>-0.093</td>
<td>-0.135</td>
</tr>
<tr>
<td>( \tau_r )</td>
<td>-0.084</td>
<td>-0.084</td>
<td>-0.084</td>
<td>-0.072</td>
<td>-0.081</td>
<td>-0.084</td>
</tr>
<tr>
<td>( C )</td>
<td>0.486</td>
<td>0.486</td>
<td>0.485</td>
<td>0.480</td>
<td>0.494</td>
<td>0.471</td>
</tr>
<tr>
<td>( Z )</td>
<td>0.446</td>
<td>0.446</td>
<td>0.445</td>
<td>0.434</td>
<td>0.441</td>
<td>0.432</td>
</tr>
<tr>
<td>( l )</td>
<td>1.265</td>
<td>1.265</td>
<td>1.266</td>
<td>1.290</td>
<td>1.269</td>
<td>1.225</td>
</tr>
<tr>
<td>( k )</td>
<td>0.389</td>
<td>0.389</td>
<td>0.387</td>
<td>0.397</td>
<td>0.390</td>
<td>0.377</td>
</tr>
<tr>
<td>( a )</td>
<td>0.055</td>
<td>0.060</td>
<td>0.055</td>
<td>0.098</td>
<td>0.049</td>
<td>0.055</td>
</tr>
<tr>
<td>( RR )</td>
<td>0.420</td>
<td>0.462</td>
<td>0.420</td>
<td>0.413</td>
<td>0.420</td>
<td>0.433</td>
</tr>
<tr>
<td>( r )</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
</tr>
</tbody>
</table>

Note. Here we increase parameters by 10%.

(a) In the third column of Table 3, in response to an increase in pension level, the following set of changes happen: consumption tax rates increases, labor income tax rates decreases, while capital income tax rates and other variables are not changed.

When a government cannot choose the level of pension benefit, it relies on consumption tax rates such that they play the role of adjusting the pension value indirectly because consumption taxes can change prices, so modify purchasing power of pension. Conversely,
when pension level is lower than the optimal level, the incentive to strengthen consumption taxation is weak. Given this association between variables, we can resolve the puzzling positive correlation between pension and consumption tax rate in OECD countries. We summarize this main finding using Proposition 3.

**Proposition 3. Externally given pension can account for the puzzle.** The optimal consumption tax rate responds positively to the pension level, accounting for a positive correlation between pension and consumption tax rate in data.

Some implications can be drawn from Proposition 3. First, this finding also helps account for the observation that when expanding welfare benefits including pension payments, most countries rely on consumption tax financing to achieve economic well-being.

Second, the tax mix involves adjustment of income taxes, too. The increased consumption tax rate at the new optimum reduces real value of wages. Hence, work incentives of young generations falls, which requires a cut in the labor income tax. As shown in Table 3, the lower labor income tax functions to perfectly neutralize the young worker's moral hazard that the higher consumption taxes boost labor supply but other variables including capital stock remain the same. The total output at the new optimum is still the same as well because labor supply and capital stock do not vary.

Third, no change in the capital income tax rate in response to parameters is consistent with Chamley’s (1986) result: for revenue-raising purposes, labor income taxes can be exploited but capital income taxes should not be used in the steady state for any configuration of parameter values. Raising revenue through capital income taxes lowers incentives to save, so it prevents capital accumulation in any case. Although the optimal capital tax rate is negative for inherently low saving incentives in our setting (to be explained more in detail later), the capital tax rate does not change in response to parameter changes.

(b) As \( n \) rises, per capita capital stock decreases. This lower level of capital accumulation diminishes total output per capita of the economy and current and future consumption in the steady-state. The result is compatible with the Solow growth model that the depreciation rate of capital and the population growth rate lower the per capita capital stock.

(c) As the relative risk aversion parameter \( \gamma \) increases, agents become more risk-averse, and thus more eagerly need consumption smoothing. In this case, the consumption in the second period becomes a more sensitive issue. Setting the consumption tax rate low improves social welfare.

(d) Next, we will see the effects of work aversion parameter \( M \). The consumption tax rate rises while the labor income tax rate goes opposite in response to an increase in \( M \). When young individuals avoid working, the efficient ways to raise work incentives is to lower the labor income tax rate while increasing the consumption tax rate.
(e) Notable is that the capital income tax rate rises only when the survival probability $\theta$ goes up. An intuition behind this is that when agents know that they will live longer for sure, then their saving incentives become ‘normal,’ and therefore giving further savings incentive is not necessary. Using our two Euler equations, we can check the difference in individuals’ saving behavior according to the survival probability.

The Euler equation of inter-temporal choice (6) can be rearranged as follows:

\[(32) \quad \beta \cdot U^2_z = \frac{U^1_c}{(1+\tau_c)} \cdot \frac{(1+\tau_c)}{(1+(1-\tau_c)\tau)}\]

The survival probability is absent in this new equation. This is simply because conditional on surviving the second period, the survival probability does not affect the intertemporal relationship directly.

In another Euler equation of intra-temporal choice (5), we can find the following relationship:

\[(33) \quad U^1_c > U^3_c, \quad (\theta \cdot U^1_c + (1-\theta) \cdot U^3_c) > (\theta' \cdot U^1_c + (1-\theta') \cdot U^3_c) \quad \text{for} \quad \theta > \theta'\]

\[(5) \quad (\theta \cdot U^1_c + (1-\theta) \cdot U^3_c) \frac{(1-\tau_w)}{(1+\tau_c)} = -(\theta \cdot U^1_l + (1-\theta) \cdot U^3_l)\]

where $U^1(\cdot)$: the first period utility conditional on surviving the second period;

$U^3(\cdot)$: the first period utility conditional on not surviving the second period.

In eq. (33-1), the first period marginal utility of consumption when surviving, $U^1_c$, is bigger than the one when not surviving, $U^3_c$, because consuming all income before dying makes $U^3_c$ lower than $U^1_c$. Thus, it is clear that when the survival probability goes higher, the weighted average of the first period marginal utility of consumption, $(\theta \cdot U^1_c + (1-\theta) \cdot U^3_c)$ becomes higher as well. An increase in the survival probability causes a rise in labor supply, as we see from the Euler equation (5) for intra-temporal choices.

We also verify that the government has incentives to foster saving of those with a low survival probability and boost capital accumulation by setting the capital income tax low, as seen from the two Euler equations combined:

\[(34) \quad \left(\theta \cdot \beta \cdot U^2_z \cdot \frac{(1+\tau_c)(1+(1-\tau_r)\tau)}{(1+\tau_c)} + (1-\theta) \cdot U^3_c \right) \frac{(1-\tau_w)}{(1+\tau_c)} = -(\theta \cdot U^1_l + (1-\theta) \cdot U^3_l)\]

When survival probability is low, reducing capital income tax elicits more labor supply from individuals, because they adjust behavior to equate both sides of eq. (34). Thus, they save more money, increasing capital stock, improving social welfares. As the survival probability gets closer to unity, labor supply, private saving, and capital accumulation reach their
maximum where the government implements a zero capital income tax rate. This summarized in Table 4.

Table 4. The Impacts of Population Aging on the Tax Structure

<table>
<thead>
<tr>
<th>Variable</th>
<th>Base Case</th>
<th>$\theta=0.85$</th>
<th>$\theta=0.90$</th>
<th>$\theta=0.95$</th>
<th>$\theta=0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_c$</td>
<td>0.185</td>
<td>0.275</td>
<td>0.424</td>
<td>0.648</td>
<td>0.939</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>-0.099</td>
<td>-0.186</td>
<td>-0.341</td>
<td>-0.587</td>
<td>-0.921</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>-0.084</td>
<td>-0.081</td>
<td>-0.069</td>
<td>-0.043</td>
<td>-0.010</td>
</tr>
<tr>
<td>$\sigma$</td>
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<td>0.482</td>
<td>0.479</td>
<td>0.481</td>
<td>0.486</td>
</tr>
<tr>
<td>$Z$</td>
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<td>0.440</td>
<td>0.433</td>
<td>0.428</td>
<td>0.425</td>
</tr>
<tr>
<td>$I$</td>
<td>1.265</td>
<td>1.276</td>
<td>1.294</td>
<td>1.316</td>
<td>1.339</td>
</tr>
<tr>
<td>$k$</td>
<td>0.389</td>
<td>0.392</td>
<td>0.398</td>
<td>0.405</td>
<td>0.412</td>
</tr>
<tr>
<td>$a$</td>
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<td>0.073</td>
<td>0.105</td>
<td>0.156</td>
<td>0.226</td>
</tr>
<tr>
<td>$RR$</td>
<td>0.420</td>
<td>0.416</td>
<td>0.413</td>
<td>0.410</td>
<td>0.409</td>
</tr>
<tr>
<td>$r$</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
</tr>
</tbody>
</table>

Note. In this table, we vary survival probability, holding $m$ constant at the base case level.

From Table 4, we see an unrealistic negative capital income tax rate. With survival probabilities below unity, agents supply labor less than the comparable labor supply at $\theta=1$. A similar interpretation goes to capital accumulation. This is another reason why the government has incentives to foster saving through a negative capital income tax rate. When the survival probability becomes unity, the capital income tax gets zero, supporting the well-known result of Chamley (1986).

Unlike Chamley’s model, the government in this model has incentives to use the capital income tax as a policy instrument to foster saving, but still this is not to finance expenditure. This result is owing to the lifetime uncertainty structure of our model.\(^{11}\) Meanwhile, in models with infinitely-living heterogeneous agents and the borrowing constraint, Aiyagari (1995) shows that in the presence of earnings shock, agents tend to save excessively for precautionary motives.\(^{12}\) In this situation, the government implements a positive capital income taxation to discourage excessive saving, which actually improves social welfare. However, in our model lifetime is finite and uncertain “downward” with a chance to live short. Although uncertainty usually raises precautionary savings motive, our particular kind of upper truncated uncertainty engenders individuals to save too little. This causes a negative capital income tax as optimum.

(f) The another interesting implication regarding the survival probability is that as it grows, the consumption tax rate increases. The economic intuition behind this result is as follows: as

\(^{11}\) In our model, lifetime uncertainty involves a shorter life span, which discourages saving.

\(^{12}\) Excessive saving lowers the first period consumption too much and makes consumption smoothing difficult, causing a welfare loss.
a group of elderly people gets larger – the survival probability can be interpreted as the level of population aging, young generation’s burden to sustain the current pensioners gets larger. Therefore, keeping the balance of burdens between young and old generations through reducing nominal pension level can improve welfare. Our calibration result supports this (see Table 5).

Table 5. Population Aging and the Optimal Pension Level

<table>
<thead>
<tr>
<th>Variable</th>
<th>Base Case</th>
<th>θ=0.85</th>
<th>θ=0.90</th>
<th>θ=0.95</th>
<th>θ=0.99</th>
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</thead>
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<tr>
<td>m</td>
<td>0.431</td>
<td>0.401</td>
<td>0.358</td>
<td>0.310</td>
<td>0.263</td>
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<tr>
<td>τw</td>
<td>-0.099</td>
<td>-0.103</td>
<td>-0.115</td>
<td>-0.141</td>
<td>-0.174</td>
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<tr>
<td>τr</td>
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<td>-0.081</td>
<td>-0.069</td>
<td>-0.043</td>
<td>-0.010</td>
</tr>
<tr>
<td>C</td>
<td>0.486</td>
<td>0.482</td>
<td>0.479</td>
<td>0.481</td>
<td>0.486</td>
</tr>
<tr>
<td>Z</td>
<td>0.446</td>
<td>0.440</td>
<td>0.433</td>
<td>0.428</td>
<td>0.425</td>
</tr>
<tr>
<td>l</td>
<td>1.265</td>
<td>1.276</td>
<td>1.294</td>
<td>1.316</td>
<td>1.339</td>
</tr>
<tr>
<td>k</td>
<td>0.389</td>
<td>0.392</td>
<td>0.398</td>
<td>0.405</td>
<td>0.412</td>
</tr>
<tr>
<td>a</td>
<td>0.055</td>
<td>0.068</td>
<td>0.087</td>
<td>0.112</td>
<td>0.138</td>
</tr>
<tr>
<td>RR</td>
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<td>0.387</td>
<td>0.343</td>
<td>0.295</td>
<td>0.250</td>
</tr>
<tr>
<td>r</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
<td>0.727</td>
</tr>
</tbody>
</table>

Note. Because of indeterminacy, we should fix one of policy variables to allow for pension to freely vary. Here, we fix the consumption tax rate at the base case level. The results are evaluated at the base case equilibrium.

However, if it is impossible to change pension level due to social rigidity against pension reform, government can indirectly reduce the pension value by increasing the consumption tax rate, since the higher tax raises the price of consumption and lowers the real value of pension. Because of this reason, the positive relationship between the survival probability and the consumption tax emerges. We summarize this finding with Proposition 4.

**Proposition 4. Aging reinforces the role of consumption taxation.** The consumption tax rate rises with aging in our model with an externally given pension level.

Population aging is often viewed as a greater political voice of the increasing elderly population, which leads to further difficulties of pension reform. Our paper suggests that in the ear of the growing elderly people, consumption taxes play a more important role, given difficulties of pension reform.

V. Conclusion
Public pension has been a central fiscal policy issue in countries with a generous welfare system that is difficult to reform. We began with highlighting the data from OECD countries that exhibit a puzzling positive correlation between pension level and the consumption tax. This paper tried to account for this puzzle in an optimal taxation framework under social rigidity against reforming pension systems. We analyzed and numerically showed the role of consumption taxation as an essential policy instrument to fix the distortions arising from inefficiencies of public pensions. Our findings also account for (i) why most countries resort to consumption tax financing when expanding welfare, and (ii) why the role of consumption taxes becomes more important with population aging.

At the very least, we believe that this paper provides a new insight into the tax policy: when expenditure-side rigidities are present, a proper tax mix can improve welfare. Finally, while we believe that our setting captures some essential features of modern fiscal policies, further research seems warranted to study the political economy regarding pensions as an endogenous element in a model.

Appendix A. Driving the Euler Equations

The Lagrangian for individual’s utility maximization problem:

\[ L(C^s_t, Z_{t+1}, l_t) = \]
\[ U(C^s_t, C^n_t, Z_{t+1}, l_t) = \theta \cdot \{ U^1(C^s_t, l_t) + \beta \cdot U^2(Z_{t+1}) \} + (1 - \theta) \cdot \{ U^3(C^n_t, l_t) \} \]
\[ + \lambda \left[ (1 - \tau_{w,t}) w_t l_t + \frac{m_{t+1}}{1 + (1 - \tau_{r,t+1}) r_{t+1}} - (1 + \tau_{c,t}) C^s_t - \frac{(1 + \tau_{c,t+1}) Z_{t+1}}{1 + (1 - \tau_{r,t+1}) r_{t+1}} \right] \]

where \( C^n_t = \frac{(1 - \tau_{w,t}) w_t l_t}{(1 + \tau_{c,t})} \).

FOCs from the optimization process are:

\[ C^s_t: \theta \cdot U^1 C^s_t - \lambda \cdot (1 + \tau_{c,t}) = 0, \]
\[ l_t: \theta \cdot U^1 l_t + (1 - \theta) \cdot U^3 l_t + \lambda \cdot (1 - \tau_{w,t}) w_t = 0. \]
\[
Z_{t+1}: \theta \cdot \beta \cdot U^2_Z - \lambda \cdot \frac{(1+\tau_{c, t+1})}{(1+1-\tau_{r, t+1})\lambda_{t+1}} = 0.
\]

Combining (A2) and (A3), the Euler equation for the intra-temporal choice between current consumption and labor is derived:

\[
(A5) \left( \theta \cdot U^1_{c,t} + (1 - \theta) U^3_{c,t} \right) \frac{(1-\tau_{w,t})w_t}{(1+\tau_{c,t})} = -(\theta \cdot U^1_{l,t} + (1 - \theta) \cdot U^3_{l,t}).
\]

Combining (A2) and (A4), the Euler equation for the inter-temporal choice between current and future consumptions is derived:

\[
(A6) \theta \cdot \beta \cdot U^2_{z,t+1} - \frac{\theta U^1_{c,t}}{(1+\tau_{c,t})} \cdot \frac{(1+\tau_{c,t+1})}{(1+1-\tau_{r,t+1})\lambda_{t+1}} = 0.
\]

References


