The Benefit Principal of Taxation and Tax Payers' Political Support

Oct, 2009
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Abstract

Based on the observation that political process is the essence of the benefit principle of taxation proposed by Wicksell and Lindahl, this paper develops a formal model incorporating the modern political system on public policy. We assume that there is a policy maker who proposes tax/public goods combination and people show their political support according to their utility from the policy. This paper provides sufficient conditions under which the benefit principle holds and efficient provision is guaranteed in political determination of public goods. In addition, even along with the principle the level of public goods can be shown to be less than the social optimum when a tax on complementary private goods should be used due to information unavailability.

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I. Introduction

According to the benefit principle of taxation, tax burden should depend on the benefit a tax payer receives from public goods of which cost government finances with the tax. Along with the ability to pay principle, it has been known as one of most important criterions to evaluate tax system. Supporters of the principle claim that if people pay taxes in line with benefits they receive, they would view them as valid price for public goods or service, not unpleasant obligation imposed by authority.

Before we go into the detail, it is helpful to classify two possible justifications on the principle. First, there has been prevailing economic literature saying that the benefit principle is a rule along which we can construct a market-like mechanism inducing efficient provision of public goods. Second, people believe that it can strengthen political acceptance of tax policy. Though the idea that tax policy is a political output is not new at all, there has not been much rigorous analysis on the principle and its implication in political economy context. This paper looks into sufficient conditions under which a political process ends up with a tax policy satisfying the principle, and analyses how it relates to efficiency of public goods provision.

Significant volume of research belong to the first category and relate the benefit principle with the optimal provision of public goods. The most famous example is Lindahl equilibrium. A simple illustration for Lindahl equilibrium is given in Figure 1. Assume that there are three agents, agent 1, 2 and 3, who consume public goods $x$. Due to non-rivality, the marginal social benefit

2) Based on common sense, not on theoretic or logical foundation, you may say that it is fair for the gainer to pay the price. It could be appealing but is not a subject of academic research.

3) One exception is Chae (2009) which models public goods provision in the context of Nash bargaining and derives a result which can be interpreted as the benefit principle.

4) For example if you look for 'benefit principle of taxation' in the Palgrave dictionary (Eatwell and Newman 1998), it just refers to 'Lindahl equilibrium' without any other content.
is the sum of individual’s marginal benefit, i.e. $MB_1 + MB_2 + MB_3$. If the marginal cost ($MC$) producing $x$ is constant, then the social optimal is obtained at $x^*$ where the marginal social benefit coincides with the marginal cost. Suppose government sets the individualized price for public goods at $f_i$ ($i=1,2,3$) and lets him/her demand as much as he/she wants. All agents would choose $x^*$ because the marginal private benefit coincides with the price there.\(^{5)}\) As $f_i$ satisfies $\sum f_i = MC$, efficient public goods provision and its finance are resolved together. The key feature of this mechanism is that individual behaves as price taker and demands (or supplies) freely as much as he/she wants given the price. In addition, government should know each agent’s marginal valuation at the targeted quantity and set up individualized price which reflects his/her marginal benefit. In the sense of setting the price at the marginal valuation, Lindahl equilibrium is claimed to follow the benefit principle. Foley(1970) extended the result and proved the existence and efficiency property of Lindahl equilibrium in a general equilibrium model.

\(^5\) The demand of agent 1 are multiple but $x^*$ surely is one of them.
By most people, a formal model of Lindahl equilibrium has been understood as a descriptive one which may be used for reference of normative evaluation. However, the original proposers such as Wicksell and Lindahl seemed to believe that the linkage between normative and operational analysis goes to the heart of this approach. The difference in perspective seems not to stem from ignorance of information (or incentive) requirement of Lindahl equilibrium, because the early proposers clearly knew that to figure out each agent's valuation is a precarious task. The real difference comes from the nature of process to be used in determining the level of public goods. The original proposers viewed the benefit principle as a product of political process while economists ever since have borrowed the private market mechanism and applied it to public goods provision.

Wicksell(1894) stated that the desired result of equating marginal costs and benefits must be achieved bia a process of negotiation, not by demand and supply.\(^6\) He viewed this process as an agreement. For this agreement to be successful, the participation should be voluntary. For the participation to be voluntary again, each agent must have a veto power, that is, unanimous voting is one to be exercised. Though he did not offer a formal model, his argument can be summarized by "unanimous voting $\Rightarrow$ voluntary participation $\Rightarrow$ negotiation process $\Rightarrow$ benefit principle $\Rightarrow$ optimal allocation".

Equilibrium in Lindahl's original work also utilizes a political process. The definition of 'equilibrium' is not a state where demand from price takers is equal to supply of government but a level of public goods every party involved agrees to be made. For example, agent 1, 2, and 3 in Figure 1 would agree unanimously that the public goods level should be \( x^* \), once \( f_i \) is their share of the cost.

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Based on the observation that collective decision making is the essence of the principle according to Wicksell and Lindahl, this paper develops a formal model incorporating the modern political system on public policy. Nowadays, large population makes it virtually impossible that all the interested parties get together and join in negotiation. Instead of this kind of direct negotiation, we assume that there is a policy proposer who proposes tax/public goods and people show their agreement or disagreement. The policy proposer maximizes political support which is a function of people's utility from public policy.

Information about agent's valuation is still a big question to overcome in any political process. As it is discussed before, Wicksell concerned much about it but seemed to believe that through voluntary negotiation it would be revealed somehow. In this paper, we consider various level of information availability, including one where the policy proposer can infer a partial information from private goods consumption.

One of main results in the paper is that the benefit principle has strong relation with efficient provision even in political determination of public goods. Suppose there are two agents, $A$ and $B$, and $A$ is a public goods consumer and $B$ isn't. Maximizing social welfare implies that the benefit of $A$ and production cost should be taken into account together. Suppose again that the policy proposer would be better off to tax all the cost on $A$. Then his decision on the level of public goods only relies on political support from the benefit of $A$ and his/her opposition to the payment of tax which is equal to the production cost. Therefore, once the benefit principle holds, that is, $A$ gets the benefit and $A$ also pays, the political process results in the same level of provision of social welfare maximization.

This model also provides sufficient conditions under which the political process leads to conforming the principle. The essential requirement, not surprisingly, is enough political power of the group with no (or little) value, which corresponds to the veto power in Wicksell's writing. Let us give an
intuitive explanation in detail. Two opposing forces are working here.

First, political support on benefit/tax structure will increase when net benefit is distributed evenly. For example suppose there are two individuals, A and B. Their benefit from public goods is 40 and 80, respectively, and the total production cost is 100. If the cost is splitted into 50 and 50, then A will vote for the policy and B will not. However if the share of cost is 30 to A and 70 to B, then both of them vote for the policy because both of their net benefit are positive. The point is that support from one voter is upper limited because he/she has at most one vote. Therefore to secure support as much as possible by dispersing the net benefit becomes the optimal strategy, and to accomplish this tax burden should be in line with the benefit.

Second, there may be tendency that tax burden leans to certain group. Suppose there are three individuals, A, B and C, and their valuation to public goods are 0, 40, 90 respectively. If the total production cost, 130, is splitted into (0 to A, 40 to B, 90 to C), then everybody's net benefit is zero. Hence they become neutral on the policy. Instead, if the burden is imposed exclusively on A, then B and C will vote for the policy though A will be against it severely. As the policy proposer counts 'neutrality' is a half, 'support' is one and 'adversity' is zero, two supports and one adversity are better than three neutrals. Since the policy proposer loses at most one vote from one individual, adversity is also limited and he would be better to exploit minor heavily. This argument tells us that a sufficient condition for the benefit principle is that everybody has enough political power so that nobody will be exploited politically.

A remark worths mentioning. There is another well known principle applied to financing government's work, that is, tax should be imposed on people who cause the public activity. For example, an agent contaminated public water in doing his/her business. When his/her community needs to clean contaminated water, the subject who messed it up at the beginning
should bear the cleaning cost. This concept can be called "Causer Principle". There is strong similarity between two as it is shown in Table 1. The gain from an activity which brought damage to others is analogous to benefit in the benefit principle. Moreover, neutrality of people with no benefit and no burden in the benefit principle is analogous to cancelling out the damage by government activity (cleaning water). Thus, we want to focus on the benefit principle, insisting that the same analysis work for "causer principle".

\[ \text{Table 1} \] The benefit principle and the causer principle

<table>
<thead>
<tr>
<th>Benefit Receiver</th>
<th>Causer</th>
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<tbody>
<tr>
<td>benefit - tax</td>
<td>gain from activity - tax</td>
</tr>
<tr>
<td>People with no benefit and not burden</td>
<td>Victim</td>
</tr>
<tr>
<td>neural</td>
<td>damage - public activity = \text{neutral}</td>
</tr>
</tbody>
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II. Model

1) Basic Set Up

Each agent faces the following utility optimization problem,

\[
\max_{x_1, z} \quad U = u(x_1) + z + \alpha g \quad \text{s.t.} \quad px + z = m \quad \text{and} \quad x = x_1 + \alpha \bar{x} - \alpha kg
\]

where \( g \) is the level of the public good the government plans to provide and \( \alpha \) is the marginal utility from public goods. \( z \) is the numeraire, \( x \) is the other private goods and its price is constant at \( p \).7) The consumption of \( x \)
consists of two, $x_1$ and $\alpha \bar{x} - \alpha kg$. $x_1$ is the amount independent of the public goods. $\alpha \bar{x}$ is the amount related with the public goods, assuming there would be no additional supply of $g$. $\alpha kg$ is the amount of the private goods to be saved by the benefit of the public goods. Lastly $m$ is the initial resource each one has.

Let us take a specific example of road. $g$ is the capacity of road that government are contemplating to construct additionally between two specific location. $\alpha$ represents frequency of commuting during the period. So $\alpha g$ is the gain in terms of time and conveniency from constructing additional road with capacity $g$. The private goods $x$ is gasoline. $x_1$ is the gasoline consumption independent of the road in question, such as usage for leisure. $\alpha \bar{x}$ is the amount of gasoline needed if there were no additional road construction, i.e. using old road. $k$ is gasoline usage per one unit of road and $\alpha kg$ is gasoline amount to be saved thanks to $g$. In sum, $\alpha \bar{x} - \alpha kg$ is gasoline consumption for commuting after the government constructs additional road of capacity $g$. Note that $x$ and $g$ are sort of complementary. Agents need both goods to order to commute, and person with higher benefit from the public goods will need the more of private goods.

The size of population is normalized one and $\alpha$ is distributed on $[0, \alpha]$ according to a distribution function $h$. $\alpha^F$ is the mean of the distribution. The production cost of $g$ is represented by $c(g)$. $c(g)$ is assumed to satisfy typical properties such as continuity, convexity, increasing in $g$ and so on, that is, $c(0) = 0$, $c'(g) > 0$, $c''(g) > 0$ and $\lim_{g \to \infty} c'(g) = \infty$. Government proposes $g$ and taxes to finance $c(g)$. Each agent shows his/her support on tax/public goods policy and it is modeled by a function $P$. $P(\bullet)$ represents political support and is interpreted as probability of casting vote 'yes' on the subject.

7) In other words, the marginal cost of producing $x$ and $z$ are constant and they are supplied in a perfectly competitive market.

8) Suppose there was a old road and the government wants to extend it.
Uncertainty (or probability) is introduced instead of binary choice of yes or no, because a lot of unobservable factors other than tax/public goods policy can influence in people's vote. Support (or probability of voting yes) increases as the proposed policy would offer higher utility. Specifically, denoting the utility from the proposed policy by $U_1$ and the utility from the current state by $U_0$, $P(\cdot)$ has the following properties.

$$0 < P(U_1 - U_0) < 1$$

$$P(0) = \frac{1}{2}, \quad \lim_{y \to -\infty} P(y) = 0, \quad \lim_{y \to \infty} P(y) = 1,$$

$$P'(y) > 0 \text{ if } y < 0, \quad P''(y) < 0 \text{ if } y > 0$$

For all $y$, $P(y) = \frac{1}{2} - P(-y)$

Most of above properties are obvious once $P(\cdot)$ is interpreted as probability. $P(y) = \frac{1}{2} - P(-y)$ means that $P$ is symmetry around $0$, which makes algebra much easier. The most interesting property is that $P$ is concave on positive and convex on negative area, as illustrated in Figure 2. If there is no uncertainty in voting, small increase of utility from status quo makes probability of voting yes 1. On the contrary, small change for the worse makes people cast no. Hence $P$ looks like one in Figure 3 in case of no uncertainty. The curvature of $P$ in Figure 2, concave on the right and convex on the left area, is a generalization of the extreme one in Figure 3.

Political support of type $\alpha$ is denoted by $P(\alpha)$ and society political support is $PS = \int_0^\alpha P(\alpha) h(\alpha) d\alpha$ which the policy proposer is assumed to maximize. To finish defining the object function of policy proposer, we need specify the

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9) Probabilistic voting model developed by Coughlin and Nitzan (1980), Coughlin et al. (1989), and Lindbeck and Weibull (1987) has a similar setup. The biggest advantage of assuming probabilistic decision on voting is that the existence of solution is guaranteed.
intestinal state. Though various cases could be studied, we pay attention to the simplest one, zero tax and zero public goods.

<Figure 2> Shape of $P$

\[ P(\cdot) \quad 1 \quad \frac{1}{2} \]

\[ U(\text{new}) - U(\text{old}) \]

<Figure 3> No Uncertainty in Voting

\[ P(\cdot) \quad 1 \quad \frac{1}{2} \]

\[ U(\text{new}) - U(\text{old}) \]

2) Benchmark: Heterogenous Lump-sum Tax

In this section, we discuss the simplest one for future reference. The policy proposer is assumed to know each agent's valuation, that is $\alpha$. In addition, there are only two types of $\alpha$ such that $\alpha_i = 0$ and $\alpha_i > 0$. The share of $\alpha_i$
and \( \alpha_h \) are \( \beta \) and \( 1-\beta \), respectively. The choice variables of the policy proposer are \( g, T_l, \) and \( T_h, \) where \( T_i \) is the lump-sum tax to agent \( \alpha_i \) \( (i = l, h). \) Of course, policy \( (g, T_l, T_h) \) should satisfy the budget constraint, 
\[
\beta T_l + (1- \beta) T_h = c(g).
\]

Keep in mind that \( g \) is not the choice variable in consumer's utility maximization but belongs to the policy proposer's proposal. Given \( (T_i, g) \), the consumer's problem will be given as follows.

\[
(x_1^*(T_i, g, \alpha_i), z^*(T_i, g, \alpha_i)) = \arg \max_{x_1, z} u(x_1) + z + \alpha_i g
\]
\[
\text{s.t. } p(x_1 + \alpha \bar{x} - kg) + z = m - T_i
\]

If \( m \) is big enough, the choice of \( x_1 \) does not depend on \( (T_i, g, \alpha_i, m) \) and is solely determined by \( u'(x_1) = p \). Let \( W \) be the maximized utility and \( W(T_i, g, \alpha_i) - W(0, 0, \alpha_i) \) be called 'policy effect of \( \alpha_i \).' Then political support given policy can be calculated as below.

\[
W(T_i, g, \alpha_i) = u(x_1^*(T_i, g, \alpha_i)) + z^*(T_i, g, \alpha_i) + \alpha_i g
\]
\[
PS = \beta P(W(T_i, g, \alpha_i) - W(0, 0, \alpha_i)) + (1- \beta) P(W(T_h, g, \alpha_h) - W(0, 0, \alpha_h))
\]

The policy proposer will maximize \( PS \) by choosing \( (g, T_l, T_h) \). The set of solution of the problem will be denoted by \( E^p \).

\[
E^p = \left\{ (T_l^p, T_h^p, g^p) | (T_l^p, T_h^p, g^p) = (\arg\max_{0 \leq T_l, T_h \leq m, g} PS(T_l, T_h, g) \right\}.
\]
\[
\text{s.t. } \beta T_l + (1- \beta) T_h = c(g)
\]

Due to the constraint of \( 0 \leq T_i \leq m \), \( g^p \) should be finite. We can easily prove that continuity and boundedness lead to \( E^p \neq \emptyset \). However since we
don't know whether concavity holds, neither uniqueness nor interiority of solution is guaranteed. Possibility of corner solution in $g$ is cumbersome without giving any additional lesson. To prevent this, we add one more assumption such that $m < c(\bar{g})$ holds where $\bar{g}$ is defined by the equation of $\alpha^E(pk+1)g = c(g)$. (See Figure 4) $\alpha^E(pk+1)g$ is the weighted sum of private benefit and $c(g)$ is the social cost. Therefore net social benefit will be maximized when $\alpha^E(pk+1) = c'(g)$ is satisfied. $g$ satisfying $\alpha^E(pk+1) = c'(g)$ will be denoted by $g^*$.

![Figure 4: The cost function of public goods](image)

It is an interesting task to compare the result of political support maximization with that of social welfare function maximization. We may consider various social welfare functions, but we will stick to the most typical one which is sum of each agent's utility as in (8). Indeed, political support $P$ may be classified as a special kind of social welfare function because it has people's utility as element. The difference between $P$ and the social welfare
function considered here is that the latter gives equal weight to everybody’s utility but the former transforms it into the probability of supportive voting.

\[
(0 \leq T_i, T_h \leq m, g) \max_{\alpha_i} \beta W(T_i, T_h, g, \alpha_i) + (1 - \beta) W(T_i, T_h, g, \alpha_h) \quad \cdots (\star)
\]

\[
s.t. \beta T_i + (1 - \beta) T_h = c(g)
\]

\(T_i\) is a lump-sum tax and does not affect consumer's behavior. Hence (and due to quasi-linearity of utility function), When \(g\) is fixed, any combination of \((T_i, T_h)\) delivering \(g\) does offer the same welfare. So, \((T_i, T_h)\) in (\(\star\)) is not determined uniquely. However it can be easily proved that \(g^*\) is a part of any solution to (\(\star\)).

Since there are only two types and \(\alpha_i = 0\), we can say that the benefit principle holds if and only if \(T_i = 0\). Proposition 1 says that if the share of the group with no benefit is as large as a half, then the benefit principle works and efficient provision of public goods is guaranteed. On the contrary, if the share of the group with no benefit is small, then the policy proposer may be better off by imposing all the burden to minor. Besides, Lemma 1 implies that a positive tax burden to no benefit people results in oversupply of public goods.

**Lemma 1:** \((T_i^p, T_h^p, g^p)\) \(\in E^p\) implies \(g^p \geq g^*\). In addition, if \(g^p > g^*\) holds, then \(T_i^p = 0\) or \(T_h^p = 0\).

**Proof:** In the appendix.

**Proposition 1:** Suppose \(\beta \geq \frac{1}{2}\). \((T_i^p, T_h^p, g^p)\) \(\in E^p\) implies \(g^p = g^*\) and \(T_i^p = 0\)

**Proof:** In the appendix.
The Figure 5 presents underlining intuition. Suppose $\beta = \frac{2}{3}$. $U_h$ represents $\alpha_h$’s utility from public goods before levying any cost. If the tax is on $h$, his utility will drop to $U_h'$. Instead, if $\alpha_i$ takes all the cost, then his utility will decrease from $U_i$ to $U_i'$. When $\beta = \frac{2}{3}$, $U_h - U_h' = 2(U_i - U_i')$. Due to the shape of $P$, $2(P(U_i) - P(U_i')) > P(U_h) - P(U_h')$. Therefore, it would be better to impose tax only to $\alpha_h$. The key point is that due to larger population of no benefit group, their policy effect is closer to zero (neutrality) and their political support is more important. Once the benefit and cost burden are concentrated on one group, it is quite natural to have the optimal level of public goods. The policy proposer will concern only support/adversity of $\alpha_h$ which is basically from his/her net utility, and the social welfare function also is composed of his/her net benefit, the total benefit minus production cost.

3) Homogeneous Lump-sum Tax and Linear Fee
In this section, we extend the previous results by allowing general distribution of $h$. Moreover, we assume that the policy proposer has to use smaller information such that homogeneous lump-sum tax and linear fee on benefit of public goods are imposed. They require much less information than individualized tax, because government needs to observe only unit service from public goods, not total benefit.

Note that the benefit of public goods is $p ak g + og$. Policy will consists of $(f, T_s, g)$, where $T_s$ is homogeneous lump-sum tax, $f$ is marginal fee and $T(\alpha) = f \times (1 + pk) og + T_s$ is the total amount of tax. To avoid unnecessary inconvenience, $0 \leq f \leq 1$ and $0 \leq T_s$ are assumed, and $W(f, T_s, g)$ and $PS$ are redefined accordingly. policy proposer’s maximization problem will be given as follows.

$$\max_{0 \leq f, 0 \leq T_s \leq m, g} \left( \int P(W(f, T_s, g, \alpha) - W(0, 0, 0, \alpha))h(\alpha) d\alpha \right)$$

$$s.t. \int f(1 + pk) og h(\alpha) d\alpha + T_s = c(g)$$

As in section 2, the policy maximizing the social welfare function is not unique but $g^*$ is a part of any of them. We need to clarify again the meaning of the benefit principle. $T_s$ is lump sum amount applied equally to everybody and does not depend on private valuation of public goods. If $T_s > 0$, an individual with $\alpha = 0$ should pay positive amount of tax despite of zero benefit from public goods. Therefore, we say that the benefit principle holds if only if $T_s = 0$.

Proposition 2: Suppose $h(\bullet)$ is symmetric, that is, $\alpha + \bar{\alpha} = 2\alpha^F \Rightarrow h(\alpha) = h(\bar{\alpha})$. If
Proof) In the appendix.

Proposition 3: Suppose there are only two types, $\alpha_l$ and $\alpha_h$ ($> \alpha_l$). In addition, assume that $\beta \geq \frac{1}{2}$ where $\beta$ is the share of $\alpha_l$ in the population. If $(f^*, T^*_p, g^*) \in E^*_p$, then $g^* = g^*$ and $T^*_p = 0$.

Proof) In the appendix.

Proposition 2 and 3 provide sufficient conditions for the benefit principle and efficient allocation of public goods. Since Proposition 3 is virtually the same as Proposition 1 in section 2, let us explain intuition of Proposition 2. Symmetry allows us to compare a unit of increase of utility of low consumer with that of high consumer located on the symmetric point of $\alpha$ horizon. Then policy effect of low consumer is smaller (and closer to zero) than that of high consumer as long as there is tax amount independent of the benefit. Therefore, the policy proposer will concern more of low consumer and the benefit principle will be attractive one.

Proposition 4 more clearly states the role of policy proposer's concern on minority. Wicksell stressed a veto power of an agent and argued that it would be a requirement of the optimal provision of public goods. In this model, an individual may get very poor treatment because all he/she can do is cast no and his/her voting may not account much. Proposition 4 says that if the policy proposer should guarantee a certain level of utility to everybody, the concern of political support from every group lets the benefit principle work and the optimal provision of public goods be achieved.

Proposition 4: Suppose the government has a constraint that for all $\alpha$, the
net benefit from the government policy, \((1-f)(1+pk)\alpha g - T_x\), should be more than \(\max \{-c(g^*), c(g^*) - (1+pk)\alpha^* g^*\}\). In addition, \(\alpha = 0\) is included in the support of \(h(\bullet)\). If \((f^*, T_x^*, g) \in E_x^p\) then, \(g^* = g^*\) and \(T_x^* = 0\).

4) Taxing on Complementary Private Goods

In this section, we will go further to see what to expect from more restriction on information available to tax authority. Here, government does tax on consumption of complementary private goods instead of any fee on public goods. Let’s take the example of road. To commute on the road by vehicle, it is inevitable to spend some kind of petroleum. In many countries gasoline tax exists and is believed to be a proxy of road charge, especially when collecting toll fee is technically difficult.\(^{10}\)

Let \((t, T_x, g_x)\) be the choice variable of policy proposer where \(t\) is per unit tax on \(x\), \(T_x\) is homogeneous lump sum tax and \(g_x\) is, of course, the level of public goods. \(W(t, T_x, g_x), PS(t, T_x, g_x)\) and \(E_x^p\) are redefined accordingly. Political support denoted by \(PS\) will be as follows.

\[
PS = \int_0^\alpha P\left(\frac{u(x_1(p+t)) - (p+t)x_1(p+t)}{-(p+t)(\alpha x - \alpha k g_x) + \alpha g_x - T_x} \right) h(\alpha) d\alpha.
\]

Policy effect are composed of three parts. \(-(p+t)(\alpha x - \alpha k g_x) + \alpha g_x - T_x\) is the change of expenditure on \(x\) including tax payment. \(u(x_1(p+t)) - (p+t)x_1(p+t) - u(x_1(p)) + px_1(p)\) is the effect on utility resulting from price change of \(x_1\). Last, \(\alpha g_x - T_x\) is the benefit from public goods minus lump sum tax. Based on the similar argument in section 3, we can say that

\(^{10}\) See Teja (1988) for this argument.
the benefit principle holds as long as $T_x = 0$.

As we already discussed, under some conditions political support suggests that $t$ should replace $T_x$. However, taxing on $x$ changes the price of consumption goods, which cause a distortion in individual's choice.\footnote{Note that the social welfare maximizing policy is to finance only through lump-sum tax, i.e. $(t, T_x, g_x) = (0, c(g^*'), g^*)$.} Because distortion means utility reduction which is also responsible for reduction in political support, policy proposer would take into account both, distortion effect and the benefit principle.

Proposition 5, 6 and 7 below which correspond to Proposition 2, 3 and 4, respectively, state that at the margin, benefit from the benefit principle is larger than that of avoiding distortion so that $t^p > 0$. Intuitively, the total deadweight loss is proportionate to tax rate, so the marginal distortion effect is zero at $t = 0$ but the marginal benefit from enforcing the benefit principle is strictly positive.

More interesting result is that once there is distortion from consumption tax, the optimal provision of public goods is also violated. $t > 0$ (or $T_x^p < c(g^p)$) implies two things. First, due to purely distortion of financing public goods, the pulic goods is less valuable than before and the policy proposer has an incentive to lower its level. Second, while $t$ fits better than $T_x$ to the benefit principle, all the agents will still bear some tax burden from $t$ because they consume $x_1$, the private goods independent of $g$. Therefore $t > 0$ carries a negative effect in terms of political support, compared to direct fee on the benefit of public goods, which induces under-supply again.

**Proposition 5:** Suppose $h(\alpha)$ is symmetric, that is, $\alpha + \hat{\alpha} = 2\alpha^F \Rightarrow h(\alpha) = h(\hat{\alpha})$. If $(t^p, T_x^p, g_x^p) \in E_x^p$, then $g_x^p < g^*$ and $t^p > 0$.

proof) In the appendix.
Proposition 6: Suppose there exist two types, \( \alpha_l \) and \( \alpha_h \) \((\alpha_h > \alpha_l)\). In addition, the share of \( \alpha_l \) is at least as large as that of \( \alpha_h \) (i.e. \( \beta \geq \frac{1}{2} \)). If \((t^*, T^*_x, g^*_x) \in E^*_x\), then \( g^*_x < g^* \) and \( t^*_x > 0 \).

proof) In the appendix.

Proposition 7: Suppose the government has a constraint that for all \( \alpha \), the net benefit from the government policy, \((1 + pk)\alpha g - T_x\), should be more than \( \max\{-c(g^*), c(g^*) - (1 + pk)\alpha g^*\} \). In addition, \( \alpha = 0 \) is included in the support of \( h(\cdot) \). If \((t^*, T^*_x, g^*_x) \in E^*_x\), then \( g^*_x < g^* \) and \( t^*_x > 0 \).

proof) In the appendix.

III. Conclusion

The benefit principle has been believed to be a virtue, making us able to build demand/supply system of public goods which resembles the market mechanism of private goods. In a competitive market, people behave as price taker, and the equilibrium price is determined through demand and supply equation.

But what the original proposers such as Wicksell and Lindahl clearly had in mind was not a mimic of the market mechanism but a political process of interested parties. For example, 'voluntary participation' in their works does not mean agent's demanding or suppling freely given a price, but literal participation in negotiation or bargaining in order to determine collectively the level of public activities.

Based on this idea, in this paper, we construct a political process which
reflects the modern policy making, analyze political decision on tax and public goods, and study the role of the benefit principle of taxation. Specifically, this paper provides sufficient conditions under which politician's maximization of political support leads to obedience of the benefit principle. Generally speaking, the condition can be interpreted as giving enough political power to the group of people with no (little) benefit from public goods and correspond to 'voluntary participation' of original proposers. Moreover as long as the principle holds, there is the linkage between efficient allocation and pursuing the maximum of political support. In this sense, the benefit principle has another reliable justification in political economy context rather than public goods' version of market system.

Furthermore, we extended the previous results to the case where government can't directly observe any information on private valuation to public goods but is able to figure it out indirectly from consumption of complementary private goods. Then taxing private goods could replace collecting fee on public goods as the typical example is petroleum tax as a proxy of road charge. However, since taxing on private goods causes distortion due to price change, politician should take into consideration of loosing some political support coming from distortion. With the same assumption under which the benefit principle would hold in a world of directly taxing public goods, it can be shown that the distortion effect does not disappear completely. The existence of distortion makes public goods less valuable to society (and to politician too), so that it results in under-provision of public goods.
Reference

Lemma 1: \((T_l^p, T_h^p, g^p) \in E^p\) implies \(g^p \geq g^*\). In addition, if \(g^p > g^*\) holds, then \(T_h^p = 0\) or \(T_l^p = 0\).

proof ① On the contrary to the conclusion, suppose \(g^p < g^*\). The policy proposer can propose \(g^*\) with the following \((T_l^p, T_h^p)\). \((i = l, h)\)

\[
T_i = T_i^p + \frac{c(g^*) - c(g^p)}{(1 + pk)\alpha_i (g^* - g^p)} (1 + pk)\alpha_i (g^* - g^p)
\]

\[
= T_i^p + \frac{\alpha_i}{\alpha} (c(g^*) - c(g^p))
\]

It is easy to see the policy above satisfies the budget constraint. Now let's compare the policy effect.

\[
(1 + pk)\alpha_i g^* - T_i^p - \frac{c(g^*) - c(g^p)}{(1 + pk)\alpha_i (g^* - g^p)} (1 + pk)\alpha_i (g^* - g^p)
\]

\[
> (1 + pk)\alpha_i g^* - T_i^p - (1 + pk)\alpha_i (g^* - g^p)
\]

\[
= (1 + pk)\alpha_i g^p - T_i^p
\]

The first inequality comes from the definition of \(c(g^*)\). The policy effect from both of \(\alpha_i\) and \(\alpha_h\) increases and it is a contradiction to \((T_l^p, T_h^p, g^p) \in E^p\).

② Suppose \(g^p > g^*\), \(T_h^p > 0\) and \(T_l^p > 0\). By the definition of \(g^*\), \(g^p > g^*\) implies that we can find a small enough \(\epsilon\) such that

\[
(1 + pk)\alpha_i \epsilon - (c(g^p) - c(g^p - \epsilon)) \frac{\alpha_i}{\alpha} < 0.
\]

Let's consider the policy of \(g^p - \epsilon\) and the following \((T_l^p, T_h^p)\). \((i = l, h)\)
\[ T_i = T_i^p - \frac{c(g^p) - c(g^p - \epsilon)}{(1 + pk)a_i} (1 + pk)a_i \epsilon \]

\[ = T_i^p - \frac{\alpha_i}{a_i^E} (c(g^p) - c(g^p - \epsilon)) \]

It is easy to see the policy above satisfies the government budget constraint. In addition, the policy effect of each type becomes bigger than that of \((T_i^p, T_h^p, g^p)\) as it can be shown below, which is a contradiction to \((T_i^p, T_h^p, g^p) \in E^p\):

\[ (1 + pk)a_i (g^p - \epsilon) - T_i^p + \frac{c(g^p) - c(g^p - \epsilon)}{(1 + pk)a_i^E} (1 + pk)a_i \epsilon \]

\[ = (1 + pk)a_i g^p - T_i^p - (1 + pk)a_i \epsilon + (c(g^p) - c(g^p - \epsilon)) \frac{\alpha_i}{a_i} \]

\[ > (1 + pk)a_i g^p - T_i^p \]

**Proposition 1:** Suppose \(\beta \geq \frac{1}{2}\). \((T_i^p, T_h^p, g^p) \in E^p\) implies \(g^p = g^*\) and \(T_i^p = 0\)

proof)  ① Let’s show \(g^p = g^*\). Suppose \(g^p \neq g^*\). By Lemma 1 it means \((g^p > g^* \text{ and } T_i^p = 0)\) or \((g^p < g^* \text{ and } T_i^p = 0)\).

step 1: Let’s show that \((g^p > g^*, T_i^p = 0)\) is not possible. \((g^p > g^*, T_i^p = 0)\) implies \(T_h = \frac{1}{1 - \beta} c(g^p)\). Fixing \(T_i^p = 0\) and putting \(T_h = \frac{1}{1 - \beta} c(g^p)\) into \(PS\) makes it a function of \(g\). Differentiating \(PS\) with respect to \(g\) at \(g^p\) gives us,

\[ \frac{dPS}{dg} \bigg|_{g=g^p} = P' \left( (1 + pk)a_i g^p - \frac{1}{1 - \beta} c(g^p) \right) (1 - \beta) (1 + pk)a_i - c'(g^p) < 0. \]

The inequality comes from the condition of \(g^p > g^*\). It means by lowering \(g\) and \(T_h\) a little bit, political support can increase, which is a contradiction.
step 2: Let's show that \((g^p > g^*, T_i^p = 0)\) is not possible. \((g^p > g^*, T_i^p = 0)\) means 
\(T_i = \frac{1}{\beta} c(g^p)\). Fixing \(T_i^p = 0\) and putting \(T_i = \frac{1}{\beta} c(g)\) into \(PS\) makes it a 
function of \(g\). Differentiating \(PS\) with respect to \(g\) at \(g^p\) gives us,

\[
\frac{dPS}{dg}_{g = g^p} = \beta P' \left( -\frac{1}{\beta} c(g^p) \right) + (1 - \beta) P'((1 + pk)\alpha_hg^p)(1 + pk)\alpha_h
\]

\[
= P' \left( -\frac{1}{\beta} c(g^p) \right) - c'(g^p) + P'((1 + pk)\alpha_hg^p)(1 - \beta)(1 + pk)\alpha_h
\]

Under \(m < c(g)\) and \(\beta \geq \frac{1}{2}\), it holds that

\[
-\frac{1}{\beta} c(g^p) + (1 + pk)\alpha_hg^p
\]

\[
= \frac{1}{\beta} ((1 + pk)\alpha_hg^p - c(g^p))
\]

\[
\geq \frac{1}{\beta} ((1 - \beta)(1 + pk)\alpha_hg^p - c(g^p))
\]

\[
= \frac{1}{\beta} ((1 + pk)\alpha_h^p - c(g^p))
\]

\[
g > 0,
\]

which means \(P' \left( -\frac{1}{\beta} c(g^p) \right) > P'((1 + pk)\alpha_hg^p)\). In addition, \(g^p > g^*\) implies 
\((1 - \beta)(1 + pk)\alpha_hg^p < c'(g^p)\). Combining these, we get \(\frac{dPS}{dg}_{g = g^p} < 0\). It means by 
lowering \(g\) and \(T_i\) a little bit together, the political support becomes larger, 
which is a contradiction.

\(2\) Now we need to show \(T_i^p = 0\). Suppose \(T_i^p > 0\) on the contrary. With \(T_i^p\) 
fixed, the budget constraint gives us \(\frac{dT_i^p}{dg} = \frac{c'(g)}{\beta}\). Using this, we can
differentiate $PS$ with respect to $g$ as follows

$$\frac{dPS}{dg}_{|g=g^*} = \beta P'(-T^p)\left(-\frac{1}{\beta}c'(g^*)\right)$$

$$+ (1-\beta)P'((1+pk)\alpha_h g^* - T^p_h)\left(1+pk\right)\alpha_h.$$  

$\beta \geq \frac{1}{2}$ implies,

$$-T^p + (1+pk)\alpha_h g^* - T^p_h$$

$$= \frac{1}{1-\beta}\left\{(1-\beta)(1+pk)\alpha_h g^* - (1-\beta)T^p - (1-\beta)T^p_h\right\}$$

$$\geq \frac{1}{1-\beta}\left\{(1-\beta)(1+pk)\alpha_h g^* - \beta T^p - (1-\beta)T^p_h\right\}$$

$$= \frac{1}{1-\beta}\left\{(1+pk)\alpha_h g^* - c(g^*)\right\} > 0.$$  

Therefore, $P'(-T^p) > P'((1+pk)\alpha_h g^* - T^p_h)$ holds. Besides, 

$$(1-\beta)(1+pk)\alpha_h g^* = c(g^*).$$  

Now we can conclude $\frac{dPS}{dg}_{|g=g^*} < 0$, which is a contradiction to $(T^p_h, T^p_h, g^*) \in E^p$.

**Lemma 2:** Suppose $h(\cdot)$ is symmetric, that is, $\alpha + \hat{\alpha} = 2\alpha^F \Rightarrow h(\alpha) = h(\hat{\alpha})$. If any real numbers $b, \lambda$ and positive numbers $a, \eta$ satisfy $a\alpha^F + b > 0$ and $\eta\alpha^F + \lambda = 0$, then

$$\int_0^\alpha P'(a\alpha + b)(\eta\alpha + \lambda)d\alpha < 0.$$  

**Proof** $a\alpha^F + b > 0$ implies that for all $s \in [0, \alpha^F]$, $a(\alpha^F - s) + b + a(\alpha^F + s) + b > 0$ holds and the symmetry of $P(\cdot)$ gives us $P'(a(\alpha^F - s) + b) > P'(a(\alpha^F + s) + b)$. Therefore,
\[
\int_0^{\alpha_E} P'(a\alpha + b)(\eta\alpha + \lambda)h(\alpha)d\alpha
\]

\[
= \int_0^{\alpha_E} P'(a(\alpha_E - s) + b)(\eta(\alpha_E - s) + \lambda)h(\alpha_E - s)ds
\]

\[
+ \int_0^{\alpha_E} P'(a(\alpha_E + s) + b)(\eta(\alpha_E + s) + \lambda)h(\alpha_E + s)ds
\]

\[
< \int_0^{\alpha_E} P'(a(\alpha_E + s) + b)(\eta(\alpha_E - s) + \lambda)h(\alpha_E - s)ds
\]

\[
+ \int_0^{\alpha_E} P'(a(\alpha_E + s) + b)(\eta(\alpha_E + s) + \lambda)h(\alpha_E + s)ds
\]

\[
= \int_0^{\alpha_E} P'(a(\alpha_E + s) + b)(\eta(\alpha_E - s) + \eta(\alpha_E + s) + 2\lambda)h(\alpha_E + s)ds = 0.
\]

Lemma 3: If any real numbers \(b, \lambda\) and any positive numbers \(a, \eta\) satisfy \(a\alpha_E + b > 0\), \(a\alpha_E + b + b > 0\) and \(\eta\alpha_E + \lambda = 0\), then \(\int_0^{\alpha_E} P'(a\alpha + b)(\eta\alpha + \lambda)h(\alpha)d\alpha < 0\)

**proof** \(a\alpha_E + b > 0\), \(a\alpha_E + b + b > 0\) and symmetry of \(P(\cdot)\) implies that for any \(s \in [0, \alpha_E]\) \(P'(as + b) > P'(a\alpha_E + b)\). Therefore,

\[
\int_0^{\alpha_E} P'(a\alpha + b)(\eta\alpha + \lambda)h(\alpha)d\alpha
\]

\[
= \int_0^{\alpha_E} P'(a\alpha + b)(\eta\alpha + \lambda)h(\alpha)d\alpha + \int_0^{\alpha_E} P'(a\alpha + b)(\eta\alpha + \lambda)h(\alpha)d\alpha
\]

\[
< \int_0^{\alpha_E} P'(a\alpha_E + b)(\eta\alpha + \lambda)h(\alpha)d\alpha + \int_0^{\alpha_E} P'(a\alpha_E + b)(\eta\alpha + \lambda)h(\alpha)d\alpha
\]

\[
< \int_0^{\alpha_E} P'(a\alpha_E + b)(\eta\alpha + \lambda)h(\alpha)d\alpha + \int_0^{\alpha_E} P'(a\alpha_E + b)(\eta\alpha + \lambda)h(\alpha)d\alpha
\]

\[
= P'(a\alpha_E + b)\int_0^{\alpha_E} (\eta\alpha + \lambda)h(\alpha)d\alpha
\]

\[
= P'(a\alpha_E + b)(\eta\alpha_E + \lambda) = 0
\]
Proposition 2: Suppose $h(\cdot)$ is symmetric, that is, $\alpha + \tilde{\alpha} = 2\alpha \Rightarrow h(\alpha) = h(\tilde{\alpha})$. If $(f^p, T^p_s, g^p) \in E^p_s$, then $g^p = g^*$ and $T^p_s = 0$.

**Proof** step 1: Let’s show that $g^p < g^*$ is not possible. Suppose, on the contrary, $g^p < g^*$. Imagine the policy $(f, g, T_s) = (\lambda - \lambda(1 - f^p)\frac{g^p}{g^*}, g^*, T^p_s)$ where 

$$
\lambda \equiv \frac{c(g^*) - T^p_s}{c(g^p) + (1 + pk)\alpha E g^* - (1 + pk)\alpha E g^p - T^p_s} < 1.
$$

It is easy to see the above policy satisfies the budget constraint. The policy effect becomes larger than before, because

$$
\left(1 - \lambda \left(1 - (1 - f^p)\frac{g^p}{g^*}\right)\right) \frac{1 + pk}{\alpha g^*} - T^p_s,
$$

$$
> (1 - f^p) \frac{g^p}{g^*} \left(1 + pk\right)\alpha g^* - T^p_s,
$$

$$
= (1 - f^p) \left(1 + pk\right)\alpha g^p - T^p_s.
$$

The policy effect of each type is larger and so is political support, which is a contradiction to $(f^p, T^p_s, g^p) \in E^p_s$.

step 2: Let’s show $(g^p > g^*, f^p > 0)$ is not possible. On the contrary, suppose $g^p > g^*$ and $f^p > 0$. $g^p > g^*$ implies that we can find small enough $\epsilon > 0$ such that $(1 + pk)\alpha E (g^p - g^*) - c(g^p - \epsilon) > (1 + pk)\alpha E g^p - c(g^p - \epsilon)$. Let’s consider a new policy of $(f, g, T_s) = (\lambda' - \lambda' \left(1 - f^p\right)\frac{g^p}{g^p - \epsilon}, g^p - \epsilon, T^p_s)$, where

$$
\lambda' \equiv \frac{c(g^p - \epsilon) - T^p_s}{c(g^p) + (1 + pk)\alpha E (g^p - \epsilon) - (1 + pk)\alpha E g^p - T^p_s} < 1.
$$

The policy effect of type $\alpha$ becomes larger because
\[
\left(1 - \lambda' + \lambda'(1 - f^p) \frac{g^p}{g^p - \epsilon}\right)(1 + pk)\alpha(g^p - \epsilon) - T^p_s \\
> (1 - f^p) \frac{g^p}{g^p - \epsilon} (1 + pk)\alpha(g^p - \epsilon) - T^p_s \\
= (1 - f^p)(1 + pk)\alpha g^p - T^p_s.
\]

Since the policy effect of all \(\alpha\) becomes larger, so does the political support, which is a contradiction to \((f^p, T^p_s, g^p) \in E^p_s\).

**step 3:** Let’s show \((g^p > g^*, f^p = 0)\) is not possible. Suppose \(g^p > g^*\) and \(f^p = 0\). Now consider a new policy of \((f, g, T_s) = (0, g^*, c(g^*))\). Under the new policy, for all \(s \in [0, \alpha^E]\), the change in policy effect of type \(\alpha^E - s\) and \(\alpha^E + s\) are,

\[\alpha^E - s \text{ type: } (1 + pk)(\alpha^E - s)(g^* - g^p) - c(g^*) + c(g^p), \quad (**)
\]

\[\alpha^E + s \text{ type: } (1 + pk)(\alpha^E + s)(g^* - g^p) - c(g^*) + c(g^p). \quad (***)
\]

By the definition of \(g^*\), the sum of (**) and (***) is positive. Besides, the symmetry of \(h\) implies that the change of political support is

\[
\Delta PS = \int_0^{\alpha^E} \left\{ P \left( (1 + pk)(\alpha^E - s)(g^* - c(g^*)) \right) h(\alpha^E - s) ds + P \left( (1 + pk)(\alpha^E + s)(g^* - c(g^*)) \right) \right\}
\]

\[
- \int_0^{\alpha^E} \left\{ P \left( (1 + pk)(\alpha^E - s)g^p - c(g^p) \right) h(\alpha^E - s) ds + P \left( (1 + pk)(\alpha^E + s)g^p - c(g^p) \right) \right\}
\]

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\[ = \int_0^{\alpha} \left\{ P \left( (1 + pk)(\alpha^E - s)g^* - c(g^*) \right) h(\alpha^E - s) \right\} ds \]
\[ + \int_0^{\alpha^E} \left\{ P \left( (1 + pk)(\alpha^E + s)g^* - c(g^*) \right) \right\} h(\alpha^E + s) ds \]
\[ = \int_0^{\alpha} \left\{ \begin{array}{c}
P \left( (1 + pk)(\alpha^E - s)g^* - c(g^*) \right) \\
- \left( P \left( (1 + pk)(\alpha^E - s)g^p - c(g^p) \right) \right) \\
- \left( P \left( (1 + pk)(\alpha^E + s)g^p - c(g^p) \right) \right) \\
- \left( P \left( (1 + pk)(\alpha^E + s)g^* - c(g^*) \right) \right)
\end{array} \right\} h(\alpha^E - s) ds. \]

As we can easily see in the graph below, the property of \( P \) guarantees that \( B - A > D - C \) and \( B + C > 0 \) implies \( P(B) - P(A) > P(D) - P(C) \).

By setting \( A, B, C \) and \( D \) as follows, we get \( \Delta PS > 0 \) because of \( (1 + pk)\alpha^E g^* - c(g^*) > 0 \) and \( (**) + (***) > 0 \). It is a contradiction to \( (f^p, T^p_s, g^p) \in E^p_s \).

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\[ A = (1 + pk)(\alpha^E - s)g^\circ - c(g^\circ), \quad B = (1 + pk)(\alpha^E - s)g^* - c(g^*), \]
\[ C = (1 + pk)(\alpha^E + s)g^\circ - c(g^\circ), \quad D = (1 + pk)(\alpha^E + s)g^* - c(g^*). \]

step 4: Combining step 1, 2 and 3, we can conclude \( g^\circ = g^* \). Now with fixed \( g^* \), let’s show that \( T^p_s = 0 \). Using the government budget constraint, \( PS \) can be converted into a function of \( f \) as follows.

\[
PS = \int_0^{\alpha} P((1 - f)(1 + pk)\alpha g^* - T_s)h(\alpha) \, d\alpha
\]
\[
= \int_0^{\alpha} P((1 - f)(1 + pk)\alpha g^* - c(g^*) + f(1 + pk)\alpha^E g^*) h(\alpha) \, d\alpha
\]

By differentiating \( PS \) with respect to \( f \), we get,

\[
\frac{dPS}{df} = \int_0^{\alpha} P'((1 - f)(1 + pk)\alpha g^* - c(g^*) + f(1 + pk)\alpha^E g^*)(1 + pk)g^*(\alpha^E - \alpha)h(\alpha) \, d\alpha > 0
\]

The inequality comes from Lemma 2. Therefore, if \( T^p_s > 0 \) then we would improve the political support by increasing \( f \) and lowering \( T^p_s \) together. Therefore, \( T^p_s = 0 \) should hold.

**Proposition 3:** Suppose there are only two types, \( \alpha_l \) and \( \alpha_h \) \((> \alpha_l)\). In addition, assume that \( \beta \geq \frac{1}{2} \) where \( \beta \) is the share of \( \alpha_l \) in the population. If \( (f^p, T^p_s, g^p) \in E^p_s \), then \( g^p = g^* \) and \( T^p_s = 0 \).

**Proof** When \( (f, T_s, g) \) is given, the political support is,
\[ PS = \beta P((1-f)(1+pk)\alpha_{g}g - T_{s}) + (1-\beta) P((1-f)(1+pk)\alpha_{h}g - T_{s}) \]
\[ = \beta P((1-f)(1+pk)\alpha_{g}g - c(g)) + f(1+pk)\alpha_{g}g \]
\[ + (1-\beta) P((1-f)(1+pk)\alpha_{h}g - c(g)) + f(1+pk)\alpha_{g}g. \]

step 1: The proof that \( g^{p} < g^{*} \) is a contradiction is the same as the step 1 of proposition 2.

step 2: We can show \((g^{p} > g^{*}, f^{p} > 0)\) is not possible by the same way as the step 2 of proposition 2. Hence it is omitted.

step 3: Let’s show that \((g^{p} > g^{*}, f^{p} = 0)\) is not possible. On the contrary, suppose \( g^{p} > g^{*} \) and \( f^{p} = 0 \). Then the political support becomes,

\[ PS = \beta P((1+pk)\alpha_{g}g^{p} - c(g^{p})) + (1-\beta) P((1+pk)\alpha_{h}g^{p} - c(g^{p})). \]

By taking the derivative of \( PS \) with respect to \( g \) at \( g^{*} \), we get

\[ \frac{dPS}{dg} \bigg|_{g=g^{*}} = \beta P'((1+pk)\alpha_{g}g^{p} - c(g^{p}))/((1+pk)\alpha_{l} - c'(g)) \]
\[ + (1-\beta) P'((1+pk)\alpha_{h}g^{p} - c(g^{p}))/((1+pk)\alpha_{h} - c'(g)). \]

Since \( g^{p} < g \) and \( \beta \geq \frac{1}{2} \), we know \((1+pk)\alpha_{g}g^{p} - c(g^{p}) + (1+pk)\alpha_{h}g^{p} - c(g^{p}) > 0 \) and it means \( P'((1+pk)\alpha_{g}g^{p} - c(g^{p})) > P'((1+pk)\alpha_{h}g^{p} - c(g^{p})). \) In addition, due to \( \beta(1+pk)\alpha_{g}^{p} + (1-\beta)\alpha_{h}^{p} < c'(g^{p}) \), we get \( \frac{dPS}{dg} \bigg|_{g=g^{*}} < 0 \), which is a contradiction.

Step 4: Through step 1 to step 3, we can conclude \( g = g^{*} \). Now let’s show \( T_{s}^{p} = 0 \) with fixed \( g^{*} \). Putting the budget constraint into \( PS \) and differentiating it with respect to \( f \) give us
\[
\frac{dPS}{df} = \beta P'((1-f)(1+pk)\alpha g^*-c(g^*)+f(1+pk)\alpha \theta g^*)(1+pk)g^*(\alpha^E-\alpha_i)
+ (1-\beta)P'((1-f)(1+pk)\alpha_\theta g^*-c(g^*)))(1+pk)g^*(\alpha^E+\alpha_\theta).
\]

\((1-f)(1+pk)(\alpha_i+\alpha_\theta)g^*-2c(g^*)+2f(1+pk)\alpha^E > 0\) implies

\[
P'((1-f)(1+pk)\alpha g^*-c(g^*)+f(1+pk)\alpha^E g^*)
> P'((1-f)(1+pk)\alpha_\theta g^*-c(g^*)+f(1+pk)g^E g^*).
\]

In addition, from \(\beta(\alpha^E-\alpha_\theta)+(1-\beta)(\alpha^E+\alpha_i) = 0\) and the above inequality, we conclude \(\frac{dPS}{df} > 0\). Therefore, if \(T_s^p > 0\), then \(PS\) can be larger by increasing \(f\) and lowering \(T_s\). Therefore, \(T_s^p=0\) should hold.

**Proposition 4:** Suppose the government has a constraint that for all \(\alpha\), the net benefit from the government policy, \((1+pk)\alpha g - T_s\), should be more than \(\max\{-c(g^*), c(g^*)-(1+pk)\alpha^E g^*\}\). In addition, \(\alpha = 0\) is included in the support of \(h(\cdot)\). If \((f^p, T_s^p, g) \in E_s^p\) then, \(g^p = g^*\) and \(T_s^p = 0\).

**proof step 1:** The proof that \(g^p < g^*\) is a contradiction is the same as the step 1 of proposition 2.

**step 2:** We can show that \((g^p > g^*, f^p > 0)\) is not possible by the same way as the step 2 of proposition 2.

**step 3:** Let's show that \((g^p > g^*, f^p = 0)\) is not possible. Suppose that \((g^p > g^*, f^p = 0)\). Then, for the agent with \(\alpha = 0\), his net benefit of public policy will be \((1+pk)\alpha g^p - T_s = -c(g^p) < -c(g^*)\). Therefore it violates the constraint and is contradictory.

**Step 4:** Through step 1 to 3, we can conclude \(g^p = g^*\). Let's fix \(g^p = g^*\) and
show $T^p_x = 0$. $PS$ can be transformed into a function of $f$ with the government budget constraint.

$$PS = \int_0^\alpha P((1-f)(1+pk)\alpha g^* - T_x)h(\alpha) \, d\alpha
= \int_0^\alpha P((1-f)(1+pk)\alpha g^* - c(g^*) + f(1+pk)\alpha Fg^*)(1+pk)g^*(\alpha - \alpha F)h(\alpha) \, d\alpha$$

With the fixed $g^*$, differentiating $PS$ with respect to $f$ gives,

$$\frac{dPS}{df} = - \int_0^\alpha P'(1-f)(1+pk)\alpha g^* - c(g^*) + f(1+pk)\alpha Fg^*)(1+pk)g^*(\alpha - \alpha F)h(\alpha) \, d\alpha.$$ 

When Lemma 3 is applied, $\frac{dPS}{df} > 0$ is derived. $\frac{dPS}{df} > 0$ means that when $T^p_x > 0$, $PS$ can increase with higher $f$. Therefore, we get $T^p_x = 0$.

**Proposition 5:** Suppose $h(\alpha)$ is symmetric, that is, $\alpha + \hat{\alpha} = 2\alpha F \Rightarrow h(\alpha) = h(\hat{\alpha})$. If $(t^p, T^p_x, g^p_x) \in E^p_x$, then $g^p_x < g^*$ and $t^p > 0$.

proof) Now, the $PS$ will be as follows given $(t, T_x, g_x)$.

$$PS = \int_0^\alpha P(u(x_1(p+t)) - (p+t)x_1(p+t) - (p+t)(\alpha x_1 - \alpha kg_x + ag_x - T_x)
\quad - u(x_1(p)) + px_1(p) + p\alpha x)
\quad \cdot h(\alpha) \, d\alpha$$

step 1: It is the same as the step 3 of proposition 2 to show that $(g^p_x > g^*, t^p = 0)$ is not possible.

step 2: Let’s show that $(g^p_x > g^*, t^p > 0)$ can’t be true. Suppose $g^p_x > g^*$ and $t^p > 0$. Then consider the policy $(t', g'_x, T'_x)$ as follows.

$$g'_x = g^p_x - \epsilon$$
\[ t' = \frac{(p+t^p)(\bar{x} - k g^p_x) - p(\bar{x} - k g^{'p} x_1) - (g^p_x - g^{'p} x_1)}{\bar{x} - k g^{'p} x_1} \]

\[ T_x = T_x^p + \theta + \mu \]

\[ \theta = u(x_1(p+t^p) - (p+t^p)x_1(p+t^p) - u(x_1(p+t^p)) + (p+t^p)x_1(p+t^p) \]

\[ \mu = c(g^{'p}) - c(g^p) + t^p x_1(p+t^p) + (t^p - t^{'p})(\alpha^{'p} \bar{x} - \alpha^p k g^p_x) - t^{'p} x_1(p+t^p) - \theta \]

Basically, \( t' \) is chosen to off-set the effect of lowering \( g \). \( \theta \) is also to cancel out utility effect from distortion in consumption \( x \). Lastly \( \mu \) is the term which makes the government budget constraint satisfied. It is easy to show that \( \mu < 0 \) implies that the policy effect will increase from \( (t^p, T_x^p, g^p_x) \) to \( (t', g^{'p}_x, T'_x) \).

The combining each definition gets us,

\[ \mu = c(g^{'p}_x) - c(g^p_x) - (1 + pk)\alpha^{'p} g^{'p}_x + (1 + pk)\alpha^p g^p_x \]

\[ + u(x_1(p+t^p)) - px_1(p+t^p) - u(x_1(p+t^p)) + px_1(p+t^p) \]

\( g^p_x > g^* \) means that we can find \( \epsilon > 0 \) such that \( c(g^{'p}_x) - c(g^p_x) - (1 + pk)\alpha^{'p} g^{'p}_x + (1 + pk)\alpha^p g^p_x < 0 \) holds. In addition, \( u(x_1(p+t)) - px_1(p+t) \) is a decreasing function in \( t \) and \( u(x_1(p+t^p)) - px_1(p+t^p) - u(x_1(p+t^p)) + px_1(p+t^p) < 0 \) holds. Therefore, we can conclude \( \mu < 0 \).

step 3: Let’s show that \( (g^p_x = g^*, t^p = 0) \) is not possible. Suppose \( (g^p_x = g^*, t^p = 0) \). Then by fixing \( t^p = 0 \), \( PS \) will be

\[ PS = \int_0^{\bar{\omega}} P((pk+1)\alpha g^* - c(g^*)) h(\alpha) d\alpha. \]

Differentiating \( PS \) with respect to \( g \) and applying Lemma 2 gives us
\[
\frac{dPS}{dg}\bigg|_{g = g^*} = \int_0^{\alpha} P'((1+pk)\alpha g^* - c(g^*))((1+pk)\alpha - c'(g^*)) h(\alpha) \, d\alpha < 0,
\]

which is a contradiction to \((t^p, T^p_x, g^p_x) \in E^p_x\).

**step 4:** Let's show \((g^p_x = g^*, \; t^p > 0)\) is contradictory. Suppose \(g^p_x = g^*\) and \(t^p > 0\). Fixing \(T^p_x\) and taking total difference of the budget constraint gives

\[
\frac{dt}{dg} = \frac{c'(g) - t\alpha^E k}{x_1(p+t) + t \frac{dx_1(p+t)}{dt} + \alpha E(\overline{x} - kg)} > \frac{c'(g) - t\alpha^E k}{x_1(p+t) + \alpha E(\overline{x} - kg)}.
\]

Then if we differentiate \(PS\) with respect to \(g\) with fixed \(T^p_x\) and apply Lemma 2, then we get,

\[
\frac{dPS}{dg}\bigg|_{g = g^*} = \int_0^{\alpha} P'(A)\left[(p+t)\alpha k + \alpha - (x_1(p+t) + \alpha(\overline{x} - kg^*)) \frac{dt}{dg}\right] h(\alpha) \, d\alpha
\]

\[
< \int_0^{\alpha} P'(A)\left[(p+t)\alpha k + \alpha - (x_1(p+t) + \alpha(\overline{x} - kg^*)) - \frac{c'(g^*) - t\alpha^E k}{x_1(p+t) + \alpha E(\overline{x} - kg^*)}\right] h(\alpha) \, d\alpha
\]

\[
< 0,
\]

where

\[
A \equiv u(x_1(p+t)) - (p+t)x_1(p+t) - (p+t)(\alpha \overline{x} - \alpha kg^*) + \alpha g^*
\]

\[-c(g^*) + tx_1(p+t) - t\alpha^E(\overline{x} - kg^*)
\]

\[-u(x_1(p)) + px_1(p) + p\alpha \overline{x}.
\]

Hence, \(t^p > 0\) implies \(PS\) can increase by lowering \(t\) and \(g\), which is a contradiction to \((t^p, T^p_x, g^p_x) \in E^p_x\).

**step 5:** Through step 1 to 4, we can conclude \(g^p_x < g_x^*\). Suppose now \(t^p = 0\).

From the budget constraint we know the following relation between \(t\) and \(T_x\).
\[
\frac{dT_x}{dt} = -x_1(p+t) - t \frac{dx_1(p+t)}{dt} - \alpha^E(x-kg)
\]

Differentiating \( PS \) with respect to \( t \) at 0 and applying Lemma 2 gives us

\[
\left. \frac{dPS}{dt} \right|_{t=0} = \int_0^{\bar{\alpha}} \alpha P'(1+pk)\alpha g^* - c(g^*) \left[ -x_1(p) - \alpha(x-kg) - \frac{dT_x}{dt} \right] h(\alpha) d\alpha
\]

\[
= -\int_0^{\bar{\alpha}} P'(1+pk)\alpha g^* - c(g^*) \left[ (\alpha - \alpha^E)(x-kg) \right] h(\alpha) d\alpha > 0.
\]

If \( t^p = 0 \), the above inequality shows that by increasing \( t \) and lowering \( T_x \), political support can be larger, which is a contradiction to \((t^p, T^p_x, g^p_x) \in E_x^p\).

**Proposition 6:** Suppose there exist two types, \( \alpha_t \) and \( \alpha_h (> \alpha_t) \). In addition, the share of \( \alpha_t \) is at least as large as that of \( \alpha_h \) (i.e. \( \beta \geq \frac{1}{2} \)). If \((t^p, T^p_x, g^p_x) \in E_x^p\), then \( g^p_x < g^* \) and \( t^p_x > 0 \).

**Proof:**

**step 1:** By the same way as the step 3 of proposition 3, we can show \((g^p_x > g^*, t^p_x = 0)\) is not possible.

**step 2:** It is the same as the step 2 of proposition 5 to show that \((g^p_x > g^*, t^p_x > 0)\) is not possible.

**step 3:** Let’s show \((g^p_x = g^*, t^p = 0)\) is not possible. Suppose \((g^p_x = g^*, t^p = 0)\) holds. \( t^p = 0 \) implies \( PS \) will be as follows.

\[
PS = \beta P((1+pk)\alpha g^* - c(g^*)) + (1-\beta)P((1+pk)\alpha_t g^* - c(g^*))
\]

By taking derivative of \( PS \) with respect to \( g \), we get
\[
\frac{dPS}{dg}\big|_{g = g^*} = \beta P^\prime\left((1 + pk)\alpha g^* - c(g^*)\right)\left((1 + pk)\alpha_1 - c'(g^*)\right)
+ (1 - \beta)P^\prime\left((1 + pk)\alpha_h g^* - c(g^*)\right)\left((1 + pk)\alpha_h - c'(g^*)\right).
\]

Since \((1 + pk)\alpha g^* - c(g^*) + (1 + pk)\alpha_h g^* - c(g^*) > 0\), \(P^\prime((1 + pk)\alpha_1 g^* - c(g^*))\) is larger than \(P^\prime((1 + pk)\alpha_h g^* - c(g^*))\). In addition, \(\beta(1 + pk)\alpha_1 + (1 - \beta)(1 + pk)\alpha_h - c'(g^*) = 0\) holds. Hence we can conclude \(\frac{dPS}{dg}\big|_{g = g^*} < 0\), which is a contradiction to \((t^p, T^p_x, g^p_x) \in E^p_x\).

step 4: Let's show \((g^p_x = g^*, t^p > 0)\) is contradictory. Suppose \(g^p_x = g^*\) and \(t^p > 0\). The budget constraint leads us to

\[
\frac{dt}{dg} = \frac{c'(g) - ta^E_k}{x_1(p + t) + t\frac{dx_1(p + t)}{dt} + \alpha^E(x - kg)} > \frac{c'(g) - ta^E_k}{x_1(p + t) + \alpha^E(x - kg)}.
\]

By taking derivative of \(PS\) with respect to \(g\), we get

\[
\frac{dPS}{dg}\big|_{g = g^*} = (1 - \beta)P^\prime(A)\left((p + t)\alpha k + \alpha_1 - (x_1(p + t) + \alpha_1(x - kg))\frac{dt}{dg}\right)
+ \beta P^\prime(B)\left((p + t)\alpha h k + \alpha_h - (x_1(p + t) + \alpha_h(x - kg))\frac{dt}{dg}\right)
= (1 - \beta)P^\prime(A)\left((p + t)\alpha k + \alpha_1 - (x_1(p + t) + \alpha_1(x - kg))\frac{c'(g) - ta^E_k}{x_1(p + t) + \alpha^E(x - kg)}\right)
+ \beta P^\prime(B)\left((p + t)\alpha h k + \alpha_h - (x_1(p + t) + \alpha_h(x - kg))\frac{c'(g) - ta^E_k}{x_1(p + t) + \alpha^E(x - kg)}\right).
\]

where \(A = u(x_1(p + t)) - (p + t)x_1(p + t) - (p + t)(\alpha_1x - \alpha_1kg^*) + \alpha_1g^* - T^p_x
- u(x_1(p)) + px_1(p) + pa_1x\)
\[ B = u(x_1(p + t)) - (p + t)x_1(p + t) - (p + t)(\alpha_h \bar{x} - \alpha_h kg^*) + \alpha_h g^* - T^p_x - u(x_1(p)) + px_1(p) + p\alpha_h \bar{x}. \]

\[ A + B > 0 \] means \( P^\prime(A) > P^\prime(B). \) In addition, \[ (1 - \beta) \left( (p + t)\alpha_k + \alpha_t - (x_1(p + t) + \alpha_t(\bar{x} - kg)) - \frac{c'(q) - t\alpha^E_k}{x_1(p + t) + \alpha^E(\bar{x} - kg)} \right) + \beta \left( (p + t)\alpha_h + \alpha_h - (x_1(p + t) + \alpha_h(\bar{x} - kg)) - \frac{c'(q) - t\alpha^E_h}{x_1(p + t) + \alpha^E(\bar{x} - kg)} \right) = 0. \]

Hence we get \( \frac{dPS}{dg} \big|_{g = g^*} < 0, \) which is a contradiction to \( (p^*, T^p_x, g^*_x) \in E^p_x. \)

**step 5:** Through step 1 to 4, we can conclude \( g^*_x < g_x. \) Suppose \( t^p_x = 0. \) From the budget constraints, we know \[ \frac{dT_x}{dt} = -x_1(p + t) - t \frac{dx_1(p + t)}{dt} - \alpha^E(\bar{x} - kg). \]

Differentiating \( PS \) with respect to \( t \) at 0 gives us \[ \frac{dPS}{dt} \big|_{t = 0} = \beta P^\prime((1 + pk)\alpha_g g^* - c(g^*)) \left[ -x_1(p) - \alpha_t(\bar{x} - kg) - \frac{dT_x}{dt} \right] + (1 - \beta) P^\prime((1 + pk)\alpha_h g^* - c(g^*)) \left[ -x_1(p) - \alpha_h(\bar{x} - kg) - \frac{dT_x}{dt} \right] = \beta P^\prime((1 + pk)\alpha_g g^* - c(g^*)) (\alpha^E - \alpha_t)(\bar{x} - kg) + (1 - \beta) P^\prime((1 + pk)\alpha_h g^* - c(g^*)) (\alpha^E - \alpha_h)(\bar{x} - kg). \]

\[ \beta(\alpha^E - \alpha_t)(\bar{x} - kg) + (1 - \beta)(\alpha^E - \alpha_h)(\bar{x} - kg) = 0 \]
and

\[ \beta(\alpha^E - \alpha_t)(\bar{x} - kg) + (1 - \beta)(\alpha^E - \alpha_h)(\bar{x} - kg) = 0, (1 + pk)\alpha_g g^* - c(g^*) + (1 + pk)\alpha_h g^* - c(g^*) > 0 \]

imply \( \frac{dPS}{dt} \big|_{t = 0} > 0, \) which is a contradiction to \( (p^*, T^p_x, g^*_x) \in E^p_x, \)

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Proposition 7: Suppose the government has a constraint that for all \( \alpha \), the net benefit from the government policy, \((1 + pk)\alpha g - T_e\), should be more than \( \max \{-c(g^*), c(g^*) - (1 + pk)\alpha F g^*\} \). In addition, \( \alpha = 0 \) is included in the support of \( h(\cdot) \). If \((t^p, T_e^p, g^p) \in E^*_e \), then \( g^p < g^* \) and \( t^p > 0 \).

**proof** step 1: We can show \((g^p_e > g^*, t^p = 0)\) is not possible by the same way as the step 3 of proposition 4.

step 2: The same way as the step 2 of proposition 5 proves \((g^p_e > g^*, t^p > 0)\) is not possible.

step 3: We can show \((g^p_e = g^*, t^p = 0)\) is not possible by the same method as the step 3 of proposition 6. The only difference is to use Lemma 3 instead of Lemma 2.

step 4: Let's show \((g^p_e = g^*, t^p > 0)\) is contradictory. Suppose \(g^p_e = g^*\) and \(t^p > 0\). From the budget constraint, we get

\[
\frac{dt}{dg} = \frac{c'(g) - t\alpha k}{x_1(p+t) + t - \alpha k} > \frac{c'(g) - t\alpha k}{x_1(p+t) + \alpha k}
\]

By differentiating \(PS\) with respect to \(g\) and applying Lemma 3, we get

\[
\left. \frac{dPS}{dg} \right|_{g = g^*} = \int_0^\alpha P'(A) \left( (p+t)\alpha k + \alpha - (x_1(p+t) + \alpha k) \right) \frac{dt}{dg} h(\alpha) d\alpha
\]

\[
< \int_0^\alpha P'(A) \left( (p+t)\alpha k + \alpha - (x_1(p+t) + \alpha k) \right) \frac{c'(g) - t\alpha k}{x_1 + \alpha k} h(\alpha) d\alpha
\]

\[
< 0
\]
where \( A \equiv u(x_1(p+t)) - (p+t)x_1(p+t) - (p+t)(\alpha \bar{x} - \alpha kg^*) + \alpha g^* \\
- c(g^*) + tx_1(p+t) - t \alpha E(\bar{x} - kg^*) \\
- u(x_1(p)) + px_1(p) + p\alpha \bar{x} \)

If \( t^* > 0 \), then political support will increase with lower \((t, g)\), which is a contraction to \((t^*, T_{x}^{p}, g^*_x) \in E_x^p\).

Step 5: Through step 1 to 4, we can conclude \( g^*_x < g_x \). Fix \( g^*_x \) and suppose \( t^* = 0 \). From the budget constraint,

\[
\frac{dT_x}{dt} = -x_1(p+t) - t \frac{dx_1(p+t)}{dt} - \alpha E(\bar{x} - kg^*).
\]

By differentiating \( PS \) with respect to \( t \) at 0 and applying Lemma 3 give us

\[
\frac{dPS}{dt} \bigg|_{t=0} = \int_0^\alpha P'((1 + pk)\alpha g_x^* - c(g_x^*)) \left[ -x_1(p) - \alpha (\bar{x} - kg_x^*) - \frac{dT_x}{dt} \right] h(\alpha) d\alpha
\]

\[
= - \int_0^\alpha P'((1 + pk)\alpha g_x^* - c(g_x^*)) (\alpha - \alpha E)(\bar{x} - kg_x^*) h(\alpha) d\alpha
\]

\[
> 0,
\]

which is a contradiction to \((t^*, T_{x}^{p}, g^*_x) \in E_x^p\).